Question 1

(a) $\int_0^{300} r(t)\, dt = 270$

According to the model, 270 people enter the line for the escalator during the time interval $0 \le t \le 300$.

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(b) $20 + \int_0^{300} (r(t) - 0.7)\, dt = 20 + \int_0^{300} r(t)\, dt - 0.7 \cdot 300 = 80$

According to the model, 80 people are in line at time $t = 300$.

$2 : \begin{cases} 1 : \text{considers rate out} \\ 1 : \text{answer} \end{cases}$

(c) Based on part (b), the number of people in line at time $t = 300$ is 80.

The first time t that there are no people in line is
$300 + \dfrac{80}{0.7} = 414.286$ (or 414.285) seconds.

1 : answer

(d) The total number of people in line at time t, $0 \le t \le 300$, is modeled by
$20 + \int_0^t r(x)\, dx - 0.7t$.

$r(t) - 0.7 = 0 \Rightarrow t_1 = 33.013298, t_2 = 166.574719$

t	People in line for escalator
0	20
t_1	3.803
t_2	158.070
300	80

The number of people in line is a minimum at time $t = 33.013$ seconds, when there are 4 people in line.

$4 : \begin{cases} 1 : \text{considers } r(t) - 0.7 = 0 \\ 1 : \text{identifies } t = 33.013 \\ 1 : \text{answers} \\ 1 : \text{justification} \end{cases}$

Question AB1/BC1 **Topic:** Modeling Rate
Max. Points: 9 **Mean Score: AB1:** 2.82; **BC1:** 4.13

What were the responses to this question expected to demonstrate?

The context of this problem is a line of people waiting to get on an escalator. The function r models the rate at which people enter the line, where $r(t) = 44\left(\frac{t}{100}\right)^3\left(1 - \frac{t}{300}\right)^7$ for $0 \le t \le 300,$ and $r(t) = 0$ for $t > 300;$ $r(t)$ is measured in people per second, and t is measured in seconds. Further, it is given that people exit the line to get on the escalator at a constant rate of 0.7 person per second and that there are 20 people in the line at time $t = 0.$ In part (a) students were asked how many people enter the line for the escalator during the time interval $0 \le t \le 300.$ A correct response demonstrates the understanding that the number of people entering the line during this time interval is obtained by integrating the rate at which people enter the line across the time interval. Thus, this number is the value of the definite integral $\int_0^{300} r(t)\,dt.$ A numerical value for this integral should be obtained using a graphing calculator. In part (b) students were given that there are always people in line during the time interval $0 \le t \le 300$ and were asked to determine the number of people in line at time $t = 300.$ A correct response should take into account the 20 people in line initially, the number that entered the line as determined in part (a), and the number of people that exit the line to get on the escalator. It was given in the problem statement that people exit the line at a constant rate of 0.7 person per second, so the number of people that exit the line to get on the escalator can be found by multiplying this constant rate times the duration of the interval, namely 300 seconds. In part (c) students were asked for the first time t beyond $t = 300$ when there are no people in line for the escalator. Because no more people join the line after $t = 300$ seconds, and people exit the line at the constant rate of 0.7 person per second, dividing the answer to part (b) by 0.7 gives the number of seconds beyond $t = 300$ before the line empties for the first time. Adding this quotient to 300 produces the answer. In part (d) students were asked when, during the time interval $0 \le t \le 300,$ is the number of people in line a minimum, and to determine the number of people in line (to the nearest whole number) at that time, with the added admonition to justify their answer. The Extreme Value Theorem guarantees that the number of people in line at time $t,$ given by the expression $20 + \int_0^t r(x)\,dx - 0.7t,$ attains a minimum on the interval $0 \le t \le 300.$ Correct responses should demonstrate that the rate of change of the number of people in line is given by $r(t) - 0.7.$ Solving for $r(t) - 0.7 = 0$ within the interval $0 < t < 300$ yields two critical points, t_1 and $t_2,$ so candidates for the time when the line is a minimum are $t = 0,$ $t_1,$ $t_2,$ and $t = 300.$ The number of people in line at times t_1 and t_2 is computed from $20 + \int_0^{t_1} r(x)\,dx - 0.7t_1$ and $20 + \int_0^{t_2} r(x)\,dx - 0.7t_2.$ The answer is the least of 20, these two computed values (to the nearest whole number), and the answer to part (b), together with the corresponding time t for this minimum value.

For part (a) see LO 3.3B(b)/EK 3.3B2, LO 3.4A/EK 3.4A2, LO 3.4E/EK 3.4E1. For parts (b) and (c), see LO 3.4A/EK 3.4A2, LO 3.4E/EK 3.4E1. For part (d) see LO 1.2B/EK 1.2B1, LO 2.3C/EK 2.3C3, LO 3.3B(b)/EK 3.3B2, LO 3.4A/EK 3.4A2, LO 3.4E/EK 3.4E1. This problem incorporates the following Mathematical Practices for AP Calculus (MPACs): reasoning with definitions and theorems, connecting concepts, implementing algebraic/computational processes, building notational fluency, and communicating.

How well did the response address the course content related to this question? How well did the responses integrate the skills required on this question?

In part (a), many responses showed understanding of the need to integrate a rate of change to find the net change in the number of people entering the line for the escalator. Some included the rate at which people left the line as part of the integrand, and some revealed a misunderstanding of the question by including the number of people in line initially in the answer. In part (b), responses generally accounted in some way for the rate that people left the line, although some failed to account for the 20 people in the line initially. In part (d), responses implied challenges in recognizing an objective function to minimize, evidenced by the use of $r(t)$ or $r'(t)$ for the rate of change (derivative) of the number of people in line instead of the use of $r(t) - 0.7$. Some responses identified the correct time for a minimum value, but fell short in justification of the minimum value. This included justifications that were incomplete by not considering the second critical point and/or not identifying the specific minimum value.

What common student misconceptions or gaps in knowledge were seen in the responses to this question?

Common Misconceptions/Knowledge Gaps	*Responses that Demonstrate Understanding*
• In part (b), omitting a differential could render an integral incorrect, as in $\int_0^{300} r(t) - 0.7 + 20$, which could be interpreted as $\int_0^{300} r(t)\, dt - 0.7 + 20$ OR $\int_0^{300} (r(t) - 0.7 + 20)\, dt$.	• The number of people in line at time $t = 300$ is given by $20 + \int_0^{300} (r(t) - 0.7)\, dt$.
• In part (c), responses indicating the line has no people at $t = \frac{80}{0.7} = 114.286$ seconds suggest a misunderstanding of how to handle the two time frames in the piecewise defined function r.	• The first time that there are no people in line is at time $t = 300 + \frac{80}{0.7} = 414.286$ seconds.
• In part (d), responses identifying the critical points as solutions to $r(t) = 0$ suggest a misinterpretation of the appropriate function to be minimized.	• The number of people in line at time t is given by $p(t) = 20 + \int_0^t (r(x) - 0.7)\, dx$. Critical points for the function are solutions to $p'(t) = r(t) - 0.7 = 0$.
• In part (d), basing a conclusion on only one zero of $r(t) - 0.7$ may represent a misconception of how to justify an absolute minimum using the candidates test (e.g., $r(t) - 0.7 = 0 \Rightarrow t = 33.0133 \Rightarrow$ The minimum is 3.803 at $t = 33.013$.)	• $r(t) - 0.7 = 0 \Rightarrow t_1 = 33.0133,\ t_2 = 166.5747$ $r(0) = 20,\ r(t_1) = 3.803,\ r(t_2) = 158.070,$ $r(300) = 80$. Therefore, the minimum is 4 people at time $t = 33.013$ seconds.

Based on your experience at the AP® Reading with student responses, what advice would you offer to teachers to help them improve the student performance on the exam?

Teachers can emphasize careful reading to distinguish, for example, the number of people that enter a line during a specified period from the number of people in line at the end of that period. One of these needs to consider the number in line at the start of the time period, while the other does not. Careful reading can also help students to make sure that they answer the specific question asked as opposed to a answering a generic problem. Does the question ask for the time the line is shortest, the number in line when it is shortest, or both?

Teachers should continue to emphasize the appropriate use of a differential to close an integral expression. With respect to graphing calculator use, teachers can coach students to store intermediate results in their calculator. This retains greater accuracy, whereas rounding intermediate results to 3 or fewer decimal places may produce a final answer that is not accurate to the required three decimal places.

What resources would you recommend to teachers to better prepare their students for the content and skill(s) required on this question?

- The *AP Calculus AB and Calculus BC Course and Exam Description* (*CED*) includes instructional resources for AP Calculus teachers to develop students' broader skills. Please see page 28 of the *CED* for examples of MPACs, questioning, and instructional strategies designed to develop the broader skill of "Justification," which was important in this question. A table of representative instructional strategies, including definitions and explanations of each, is included on pages 33–37 of the *CED*. The strategy "Critique reasoning," for example, may be helpful in developing students' reasoning and communication skills at multiple points across the curriculum, including when learning to use a candidates test to justify a conclusion about an absolute minimum value.
- AP Central's course pages for AP Calculus AB and AP Calculus BC include a "Classroom Resources" tab, where you will find a variety of teaching modules developed by the College Board over the years. For this question, you might refer to the module *Extrema*, which includes commentary, related previous AP Exam questions, student worksheets, teacher notes, and teaching examples.
- AP Central's course pages for AP Calculus AB and AP Calculus BC include "The Exam" tab, where you (and your students) will find questions from previous AP Exams and reflections of chief readers. In addition to end-of-course review, these resources are extremely useful for low-stakes, formative assessment, from which you may base high-quality feedback and responsive instruction. Careful study of 2017 question AB2 part (a), for example, would have been excellent preparation for 2018 question AB1/BC1 part (a). Consistently holding students accountable for clear mathematical communication, including attention to parentheses and placement of differentials, is the best way to develop such skills.
- AP Central's course pages for AP Calculus AB and AP Calculus BC include a "Professional Development" tab, where you will find:
    - Links to Online Modules for Teaching and Assessing AP Calculus. The module *Justifying Properties and Behaviors of Functions Using Derivatives* would be relevant to this question.
    - A link to Davidson Next, an initiative that, "aims to supplement Advanced Placement (AP) instruction with online modules designed for in-class, blended instruction."
- Finally, the Online Teacher Community is a great place to ask a question, share a strategy, or hear from other AP teachers.