Mean Value Theorem and 2nd FTC Worksheet Name: Answer Key

1. Find the derivatives of the functions defined by the following integrals:

(a) $\int_0^x \frac{\sin t}{t}\,dt$

$\frac{\sin x}{x}$

(b) $\int_0^x e^{-t^2}\,dt$

e^{-x^2}

(c) $\int_1^{\cos x} \frac{1}{t}\,dt$

$\frac{1}{\cos x}(-\sin x)$

(d) $\int_0^1 e^{\tan^2 t}\,dt$

0

(e) $\int_x^{x^2} \frac{1}{2t}\,dt, x>0$

$\int_x^a \frac{1}{2t}\,dt + \int_a^{x^2} \frac{1}{2t}\,dt$

$-\int_a^x \frac{1}{2t}\,dt + \int_a^{x^2} \frac{1}{2t}\,dt$

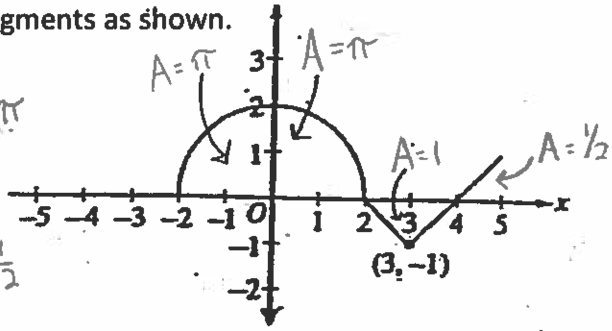
$\boxed{-\frac{1}{2x} + \frac{1}{2x^2}}$

(f) $\int_x^2 \cos(t^2)\,dt$

$-\int_2^x \cos(t^2)\,dt$

$-\cos(x^2)$

2. The graph of a function f consists of a semicircle and two line segments as shown. Let g be the function given by $g(x)=\int_0^x f(t)\,dt$.



(a) Find $g(0), g(3), g(-2)$, and $g(5)$.

$g(0)=\int_0^0 f(t)dt = 0$ $\quad g(3)=\int_0^3 f(t)dt = \pi-\frac{1}{2}$

$g(-2)=\int_0^{-2} f(t)dt = -\pi$

$g(5)\int_0^5 f(t)dt = \pi-\frac{1}{2}$

(b) Find all values of x on the open interval $(-2,5)$ at which g has a relative maximum. Justify your answers.

$g'(x)=\frac{d}{dx}\int_0^x f(t)dt$

$g'(x)=f(x)$

$0=f(x)$

$x=2, x=4$ critical points

There is a rel. max at $x=2$ b/c $g'(x)=0$ and $g'(x)$ changes from pos. to negative at that point.

(c) Find the absolute minimum value of g on the closed interval $[-2,5]$ and the value of x at which it occurs. Justify your answer. The absolute minimum of g occurs at the critical points or the endpoints. $g(-2)=-\pi$ is the absolute minimum.

$g(-2)=-\pi \quad g(2)=\pi \quad g(4)=\pi-1 \quad g(5)=\pi-\frac{1}{2}$

3. Given f(x) = $\int_{-2}^{x^2}\cos(t^2)\,dt$. Determine the second derivative of f.

$f'(x)=\cos(x^4)\cdot 2x$

$f''(x)=\cos(x^4)\cdot 2 + 2x(-\sin(x^4)\cdot 4x^3)$

$\boxed{f''(x)=2\cos(x^4)-8x^4\sin(x^4)}$

4. The velocity of a car accelerating from a red light is given by $v(t) = \frac{6}{5}x^2 + \frac{1}{2}x$. Find the average velocity of the car for the first 6 seconds after the light turns green.

$$\frac{1}{6-0}\int_0^6 \left(\frac{6}{5}x^2 + \frac{1}{2}x\right)dx \longrightarrow \frac{1}{6}\left[\frac{6}{15}x^3 + \frac{1}{4}x^2\right]_0^6$$

$$\frac{1}{6}\left[\frac{6}{15}(6)^3 + \frac{1}{4}(6)^2\right] - \frac{1}{6}[0]$$

$$\boxed{15.9}$$

5. Find the average value of the functions over the given intervals.

a) f(x) = 9 – x² [-3, 3]

$$\frac{1}{3-{}^{-}3}\int_{-3}^{3}(9-x^2)dx$$

$$\frac{1}{6}\left[9x - \frac{1}{3}x^3\right]_{-3}^{3}$$

$$\frac{1}{6}[27-9] - \frac{1}{6}[-27+9]$$

$$\boxed{6}$$

b) f(x) = x³ [0, 1]

$$\frac{1}{1-0}\int_0^3 x^3\,dx$$

$$\frac{1}{4}x^4\Big|_0^1$$

$$\frac{1}{4} - 0$$

$$\boxed{\frac{1}{4}}$$

c) f(x) = sinx [0, π]

$$\frac{1}{\pi-0}\int_0^\pi \sin x\,dx$$

$$\frac{1}{\pi}\left[-\cos x\right]_0^\pi$$

$$\frac{1}{\pi}[-\cos\pi] - \frac{1}{\pi}[-\cos 0]$$

$$\frac{1}{\pi} - \left(\frac{-1}{\pi}\right) = \boxed{\frac{2}{\pi}}$$

6. (Calculator) A zoo sponsored a one-day contest to name a new baby elephant. Zoo visitors deposited entries in a special box between noon (t = 0) and 8 p.m. (t = 8). The number of entries in the box t hours after noon is modeled by a differentialbe function E for 0 ≤ t ≤ 8. Values of E(t), in hundreds of entries, at various times t are shown in the table below.

t (hours)	0	2	5	7	8
$E(t)$ (hundreds of entries)	0	4	13	21	23

(a) Use the data in the table to approximate the rate, in hundreds of entries per hour, at which entries were being deposited at time t = 6. Show the computations leading to your answer.

$$\frac{E(7)-E(5)}{7-5} = \frac{21-13}{7-5} = \frac{8}{2} = 4 \text{ hundred entries per hour}$$

(b) Use a trapezoidal sum with four subintervals given by the table to approximate the value of $\frac{1}{8}\int_0^8 E(t)dt$. Using correct units, explain the meaning of $\frac{1}{8}\int_0^8 E(t)dt$ in terms of the number of entries.

$$\frac{1}{8-0}\left[\frac{0+4}{2}(2) + \frac{4+13}{2}(3) + \frac{13+21}{2}(2) + \frac{21+23}{2}(1)\right]$$

$$\frac{1}{8}[85.5] = 10.688 \text{ hundred entries}$$

$\frac{1}{8}\int_0^8 E(t)dt$ is the average number of entries in hundreds in the box between noon and 8pm.

(c) At 8 p.m., volunteers began to process the entries. The processed the entries at a rate modeled by the function P, where P(t) = t³ – 30t² + 298t – 976 hundreds of entries per hour for 8 ≤ t ≤ 12. According to the model, how many entries had not yet been processed by midnight?

$$23 - \int_8^{12} P(t)dt = 23 - 16 = 7 \text{ hundred entries}$$