**Calculus 1 – Sec 7.2**
**Volumes of Revolution**
**Disk Method Worksheet**

**Name**________________________________

## Find the Volumes of Revolution:

1) $y = \sqrt{x}$, $x = 1$, $x = 4$, $y = 0$ about the x-axis

2) $y = -x + 1$, $y = 0$, $x = 0$ about the x-axis

3) $y = 4 - x^2$, $y = 0$, $x = 0$, (in the first quadrant)
   a) about the x-axis
   b) about the y-axis

4) $y = x^2$, $x = 0$, $y = 4$, (in the first quadrant)
   a) about the y-axis
   b) about $y = 4$

5) $y = \sqrt{4 - x^2}$, $y = 0$, $x = 0$, (in the first quadrant) about the x-axis

6) $x = 4y - y^2$, above $y = 1$, $x = 0$, about the y-axis

7) $y = x^{\frac{2}{3}}$, $y = 1$, $x = 0$, about the y-axis

8) $y = 5x - x^2$, $y = 0$, about the x-axis

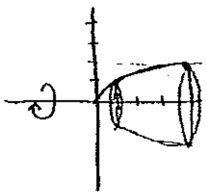
9) $y = \frac{x^2}{2}$, $y = 8$, about $y = 8$

10) $x = \sqrt{y}$, $x = 9$, $y = 0$, about $x = 9$

11) $y = \sqrt{\cos x}$, $y = 0$, $\frac{-\pi}{2} \le x \le \frac{\pi}{2}$ about the x-axis

# Volumes of Revolution:

① 



$V = \int_1^4 \pi\left((\sqrt{x})\right)^2 dx$

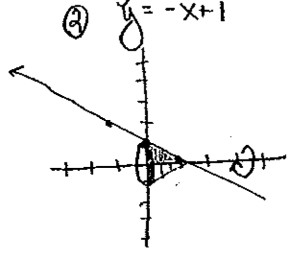
$\pi \int_1^4 x\, dx$

$= \pi \frac{x^2}{2}\Big|_1^4$

$= \pi\left(8 - \frac{1}{2}\right)$

$= \boxed{\frac{15\pi}{2}}$

② $x=0$, $y=0$, $y=-x+1$



$V = \int_0^1 \pi(-x+1)^2\, dx$

$\pi \int_0^1 x^2 - 2x + 1\, dx$

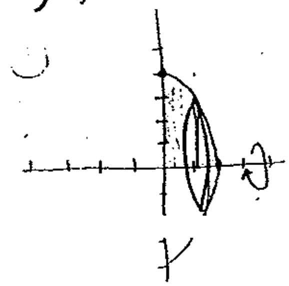
$= \pi\left(\frac{x^3}{3} - x^2 + x\right)\Big|_0^1$

$= \pi\left(\frac{1}{3} - 1 + 1\right) - 0$

$= \boxed{\frac{\pi}{3}}$

3)

a) $y = 4 - x^2$, $y = 0$, $x = 0$



$V = \pi \int_0^2 (4 - x^2)^2 dx$

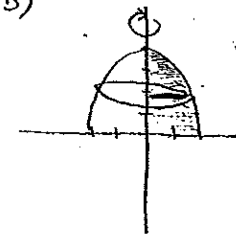
$= \pi \int_0^2 (16 - 8x^2 + x^4) dx$

$= \pi\left[16x - \frac{8}{3}x^3 + \frac{x^5}{5}\right]_0^2$

$= \pi\left[32 - \frac{64}{3} + \frac{32}{5}\right] - 0$

$= \frac{256\pi}{15}$

3 b)



$y = 4 - x^2$

$x^2 = 4 - y$

$x = \sqrt{4-y}$

$V = \int_0^4 \pi\left((4-y)^{1/2}\right)^2 dy$

$= \pi \int_0^4 4 - y\, dy$

$= \pi \cdot 4y - \frac{y^2}{2}\Big|_0^4$

$= \pi(16 - 8) - 0$

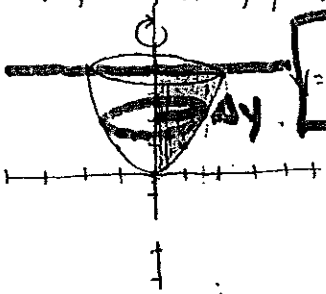
$= \boxed{8\pi}$

$f(x) = 4$

$V = \pi \int_0^0 \square^2 dx$

$\pi \int R^2\, dx/y$

4) a) $x = \sqrt{y}$, $y = x^2$, $x = 0$, $y = 4$



$dy$

$\boxed{V = \int_0^4 \pi (y^{1/2})^2 dy}$

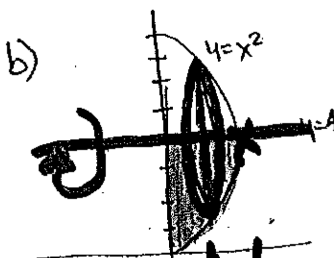
$\pi \int_0^4 y\, dy$

$= \pi \frac{y^2}{2}\Big|_0^4$

$= \pi(8 - 0)$

$= \boxed{8\pi}$

b)



$x = \sqrt{4}$

$y = x^2$

$V = \pi \int_0^2 (x^2 - 4)^2 dx$

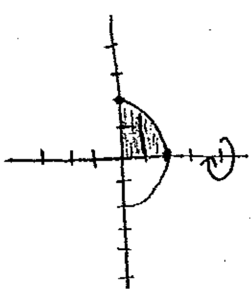
$= \pi \int_0^2 (x^4 - 8x^2 + 16) dx$

$= \pi\left[\frac{x^5}{5} - \frac{8}{3}x^3 + 16x\right]_0^2$

$= \pi\left[\frac{32}{5} - \frac{64}{3} + 32\right] - 0$

$y^2 - 4y = x$ $= \boxed{\frac{256\pi}{15}}$

#5 $y = \sqrt{4 - x^2}$, $y = 0$, $x = 0$
about x-axis



$V = \int_0^2 \pi\left[(4 - x^2)^{1/2}\right]^2 dx$

$= \pi \int_0^2 4 - x^2\, dx$

$= \pi\left[4x - \frac{x^3}{3}\right]_0^2$

$= \pi\left[8 - \frac{8}{3}\right] - 0$

$= \boxed{\frac{16\pi}{3}}$

⑥ $y = 4x - x^2$

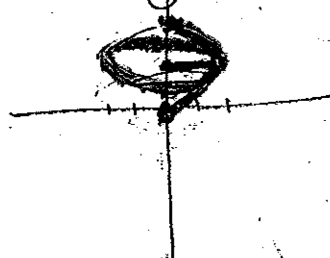
$x = 4y - y^2$ $y = 1$ $x = 0$

about y-axis

$4y - y^2 = 0$

$y(4 - y) = 0$

$y = 0, 4$



$V = \int_1^4 \pi (4y - y^2)^2 dy$

$\pi \int_1^4 16y^2 - 8y^3 + y^4\, dy$

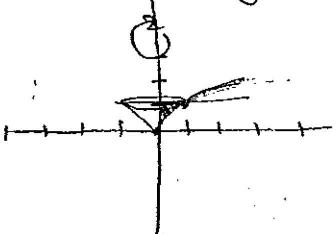
$= \pi\left[\frac{16}{3}y^3 - 2y^4 + \frac{y^5}{5}\right]_1^4$

$= \pi\left[\frac{16}{3} \cdot 64 - 512 + \frac{1024}{5}\right] - \left[\frac{16}{3} - 2 + \right.$

$= \pi\left[\frac{1024}{3} - \quad + \right.$

$= \pi\left[\frac{512}{15}\right] - \left[\frac{53}{15}\right] = \boxed{\frac{153\pi}{5}}$

$y = x^{2/3}$, $y = 1$, $x = 0$ about $y$-axis



$$V = \int_0^1 \pi \left(y^{3/2}\right)^2 dy$$

$$= \pi \int_0^1 y^3 \, dy$$

$$= \pi \frac{y^4}{4}\Big|_0^1$$

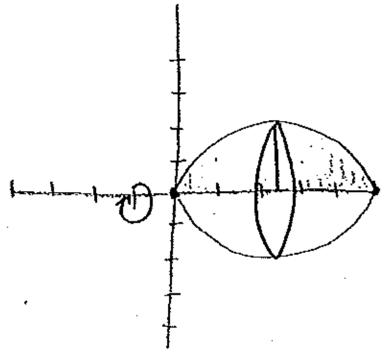
$$= \boxed{\frac{\pi}{4}}$$

$$\left(x^{2/3}\right)^{3/2} = y^{3/2}$$

$$x = y^{3/2}$$

⑧ $y = 5x - x^2$, $y = 0$ about $x$-axis

$x(5-x) = 0$



$$V = \int_0^5 \pi (5x - x^2)^2 \, dx$$

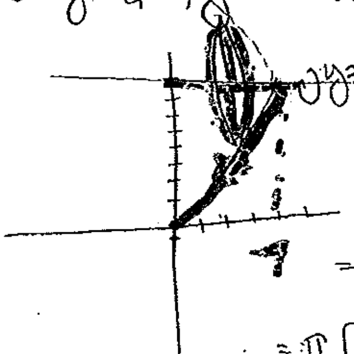
$$= \pi \int_0^5 25x^2 - 10x^3 + x^4 \, dx$$

$$= \pi \left[ 25 \cdot \frac{x^3}{3} - \frac{5x^4}{2} + \frac{x^5}{5} \right]_0^5$$

$$= \pi \left[ \frac{3125}{3} - \frac{3125}{2} + \frac{3125}{5} \right] - 0$$

$$= \boxed{\frac{625\pi}{6}}$$

⑨ $y = \frac{x^2}{2}$, $y = 8$ about $y = 8$



$$V = \int_0^4 \pi \left(8 - \frac{x^2}{2}\right)^2 dx$$

$$V = \pi \int_0^4 64 - 8x^2 + \frac{x^4}{4} \, dx$$

$$= \pi \left[ 64x - \frac{8}{3}x^3 + \frac{x^5}{20} \right]_0^4$$

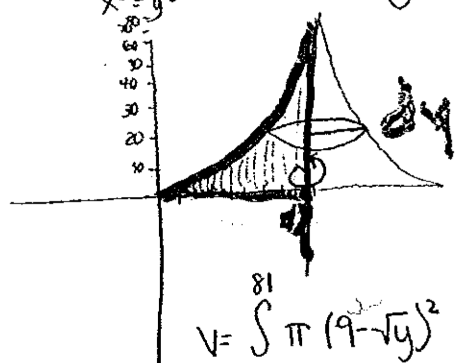
$$= \pi \left[ 256 - \frac{512}{3} + \frac{1024}{20} \right] - 0$$

$$= \pi \left[ \frac{2048}{15} \right]$$

$$= \frac{2048\pi}{15}$$

⑩ $x = \sqrt{y}$, $x = 9$, $y = 0$ about $\boxed{x = 9}$ → $9 - f(x)$

$x^2 = y$



$$V = \int_0^{81} \pi \left(9 - \sqrt{y}\right)^2 dy$$

$$= \pi \int_0^{81} \left(81 - 18y^{1/2} + y\right) dy$$

$$= \pi \left[ 81y - 12y^{3/2} + \frac{y^2}{2} \right]_0^{81}$$

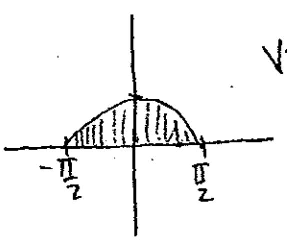
$$= \pi \left[ 6561 - 8748 + \frac{6561}{2} \right] - 0$$

$$= \pi \left[ -2187 + \frac{6561}{2} \right]$$

$$= \pi \left[ \frac{-4374}{2} + \frac{6561}{2} \right]$$

$$= \boxed{\frac{2187}{2}\pi}$$

⑪ $y = \sqrt{\cos x}$ $y = 0$ $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$



$$V = \int_{-\pi/2}^{\pi/2} \pi \left(\sqrt{\cos x}\right)^2 dx$$

$$= \pi \int_{-\pi/2}^{\pi/2} \cos x \, dx$$

$$= \pi \cdot \sin x \Big|_{-\pi/2}^{\pi/2}$$

$$= \pi \left(1 - (-1)\right)$$

$$= \boxed{2\pi}$$