SUPER SECRET NUMBER PUZZLE
IMPLICIT DIFFERENTIATION

Find the answer to each question.
Write your answer in the answer blank to the left.
Add up all of your answers.
Check to see if your number matches the super secret number!

-4 1. Find the value of the derivative of $x^3 + xy - 2 = 0$ at the point (1, 1).

-1 2. Find the value of the derivative of $x^3 + y^3 = 6xy$ at (3, 3).

2 3. What is the y-intercept of the tangent line to $x^3 + y^3 = 8$ at the point (0, 2)?

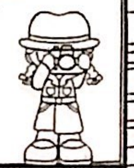
-2 4. Find the value of the second derivative of $x = y^2$ at the point where $y = \frac{1}{2}$.

-5 5. Find the slope of the normal line to the curve $x^2 - xy + y^2 = 7$ at the point where $x = 1$. (There are two possible answers - write the smaller answer on the line.)

0 6. The slope of the tangent to the curve $x^3 - 6xy - ky^3 = a$ is -1 at the point (0, 1). Find the sum of k and a.

2 7. Consider the curve $x^2 + 4y^2 = 7 + 3xy$. There is a horizontal tangent line to this curve at a point where $x = 3$. What is the y-coordinate of this point?

-2 8. Consider the curve $x^2 + 2x + y^4 + 4y = 5$. Find the sum of the x-coordinates of the two points on the curve where the line tangent to the curve is vertical.



THE SUPER SECRET NUMBER IS -10

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① $x^3 + xy - 2 = 0 \quad @ \; (1,1)$

$$3x^2 + x\frac{dy}{dx} + y = 0$$

$$3 + \frac{dy}{dx} + 1 = 0$$

$$\frac{dy}{dx} = -4$$

② $x^3 + y^3 = 6xy \quad @ \; (3,3)$

$$3x^2 + 3y^2\frac{dy}{dx} = 6\left[x\frac{dy}{dx} + y\right]$$

$$27 + 27\frac{dy}{dx} = 6\left[3\frac{dy}{dx} + 3\right]$$

$$9\frac{dy}{dx} = -9$$

$$\frac{dy}{dx} = -1$$

③ $x^3 + y^3 = 8$ at $(0,2)$

$$3x^2 + 3y^2 \frac{dy}{dx} = 0$$

$$0 + 12 \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = 0$$

$$y - 2 = 0(x - 0)$$

$$y = 2$$

④ $x = y^2$ $\qquad y = \frac{1}{2}$ $\qquad x = \frac{1}{4}$

$$1 = 2y \frac{dy}{dx}$$

$$\frac{1}{2}y^{-1} = \frac{1}{2y} = \frac{dy}{dx}$$

$$-\frac{1}{2}y^{-2} \frac{dy}{dx} = \frac{d^2y}{dx^2}$$

$$\frac{-1}{2y^2} \cdot \frac{1}{2y} = \frac{d^2y}{dx^2}$$

$$\left.\frac{d^2y}{dx^2}\right|_{y=\frac{1}{2}} = \frac{-1}{4\left(\frac{1}{2}\right)^3}$$

$$\left.\frac{d^2y}{dx^2}\right|_{y=\frac{1}{2}} = -2$$

(5) $x^2 - xy + y^2 = 7$ @ $x=1$ $1-y+y^2=7$

$y^2-y-6=0$

$(y-3)(y+2)=0$

$y=3,-2$

$$2x - \left[x\frac{dy}{dx} + y\right] + 2y\frac{dy}{dx} = 0$$

$$2x - x\frac{dy}{dx} - y + 2y\frac{dy}{dx} = 0$$

$$-x\frac{dy}{dx} + 2y\frac{dy}{dx} = -2x + y$$

$$\frac{dy}{dx} = \frac{-2x+y}{-x+2y}$$

$$\left.\frac{dy}{dx}\right|_{(1,3)} = \frac{-2+3}{-1+6} = \frac{1}{5}$$ normal line -5

$$\left.\frac{dy}{dx}\right|_{(1,-2)} = \frac{-2-2}{-1-4} = \frac{-4}{-5} = \frac{4}{5}$$ normal line $-\frac{5}{4}$

(6) $x^3 - 6xy - ky^3 = a$ $\qquad \left.\frac{dy}{dx}\right|_{(0,1)} = -1$

$k + a = ?$

$3x^2 - 6\left[x\frac{dy}{dx} + y\right] - k3y^2\frac{dy}{dx} = 0$

$-6[0+1] - k3(1)(-1) = 0$

$-6 + 3k = 0$

$k = 2$

$0 - 0 - 2 \cdot 1 = a$ $\qquad k + a = 0$

$-2 = a$

(7) $2x + 8y\frac{dy}{dx} = 0 + 3\left[x\frac{dy}{dx} + y\right]$

HTL
$x=3$
$y=?$

$$\frac{dy}{dx}\left[8y - 3x\right] = -2x + 3y$$

$$\frac{dy}{dx} = \frac{-2x+3y}{8y-3x}$$

$$-2x + 3y = 0$$

$$-6 + 3y = 0$$

$$3y = 6$$

$$y = 2$$

(8) $2x + 2 + 4y^3 \frac{dy}{dx} + 4\frac{dy}{dx} = 0$

$$\frac{dy}{dx}\left[4y^3 + 4\right] = -2x - 2$$

$$\frac{dy}{dx} = \frac{-2x-2}{4y^3+4}$$

$$4y^3 + 4 = 0$$

$$4(y^3+1) = 0$$

$$(y+1)(y^2 - y + 1) = 0$$

$y = -1$ $\quad$ $y =$ imaginary values

$$x^2 + 2x + 1 - 4 = 5$$

$$x^2 + 2x - 8 = 0$$

$(x+4)(x-2) = 0$ $\quad$ $x = -4, 2$ $\quad$ sum -2