AP Calculus AB Name _____
Cross Sections Worksheet

1. The base of a solid is bounded by $y = \cos(x)$, the x-axis, $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$. Cross sections perpendicular to the x-axis are squares. Find the volume.

2. The base of a solid is bounded by $y = 2 - x$, the x-axis, and the y-axis. Cross sections that are perpendicular to the x-axis are isosceles right triangles with the right angle on the x-axis. (Legs perpendicular to the x-axis). Find the volume.

3. The base of a solid is bounded by the semi-circle $y = \sqrt{4 - x^2}$ and the x-axis. Cross sections that are perpendicular to the x-axis are squares. Find the volume.

4. The base of a solid is bounded by $y = \sqrt{16 - x^2}$ and the x-axis. Cross sections that are perpendicular to the y-axis are equilateral triangles. Find the volume.

5. The base of a solid is a circular region in the xy-plane bounded by the graph $x^2 + y^2 = 9$. Find the volume of the solid if every cross section by a plane normal to the x-axis is an equilateral triangle with one side as the base.

6. The base of a solid is circular region in the xy-plane bounded by the graph of $x^2 + y^2 = 9$. Find the volume of the solid if every cross section by a plane normal to the x-axis is a square with one side as the base.

7. The base of a solid is bounded by $y = 2 - \frac{1}{2}x$, the x-axis, and the y-axis. Cross sections that are perpendicular to the y-axis are isosceles right triangles with the hypotenuse in the xy-plane. Find the volume.

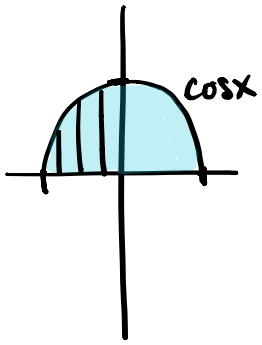
Answers

1. $\frac{\pi}{2}$
2. $\frac{4}{3}$
3. $\frac{32}{3}$
5. $36\sqrt{3}$
6. 144
7. $\frac{8}{3}$

4. $\frac{128\sqrt{3}}{3}$

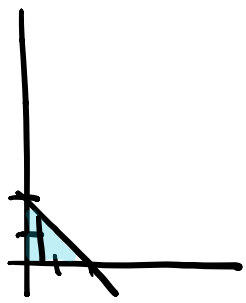
① $y = \cos(x)$

$$\int_{-\pi/2}^{\pi/2} (\cos x)^2 \, dx = \frac{\pi}{2}$$ * radian mode!



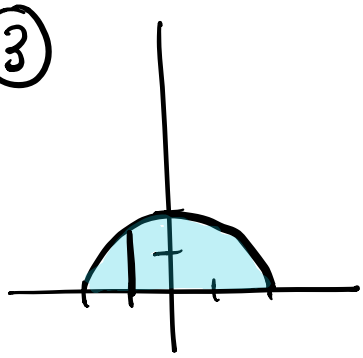
②

$$\int_0^2 \frac{1}{2}(2-x)^2 \, dx = \frac{4}{3}$$

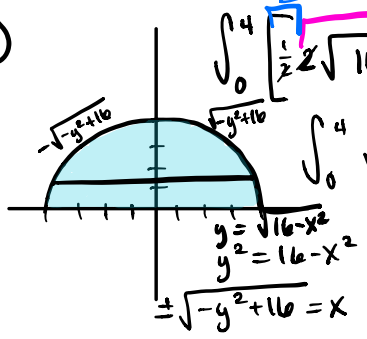


③

$$\int_{-2}^{2} \left(\sqrt{4-x^2}\right)^2 dx = \frac{32}{3}$$

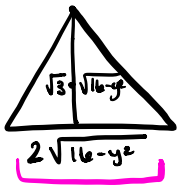


④

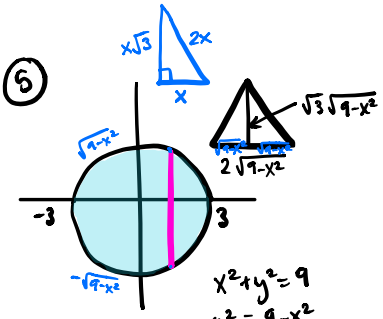


$\int_0^4 \left[\frac{1}{2}\cdot 2\sqrt{16-y^2}\right)\sqrt{3}\sqrt{16-y^2}\,dy$ (b, h)

$\int_0^4 \sqrt{3}(16-y^2)\,dy = \frac{128\sqrt{3}}{3}$



⑤



$x^2+y^2=9$

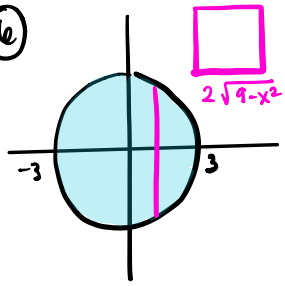
$y^2=9-x^2$

$y=\pm\sqrt{9-x^2}$

$\int_{-3}^{3}\sqrt{3}\,(9-x^2)\,dx$

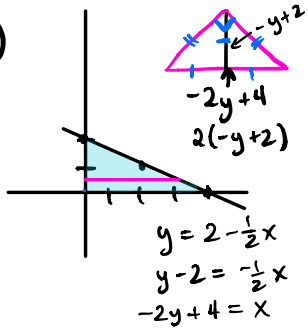
$=36\sqrt{3}$

⑥

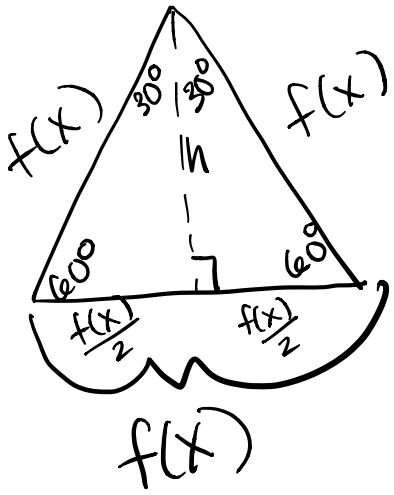


$\int_{-3}^{3}(2\sqrt{9-x^2})^2\,dx = 144$

⑦



$\int_0^2 \frac{1}{2}\cdot 2(-y+2)^2\,dy = \frac{8}{3}$

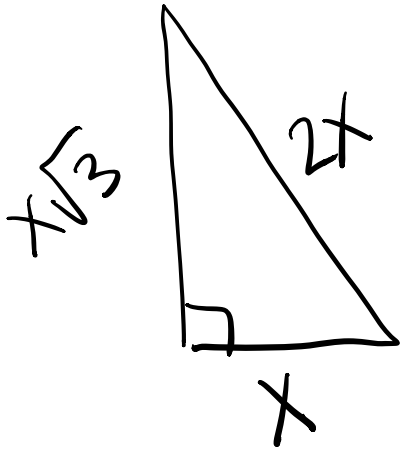


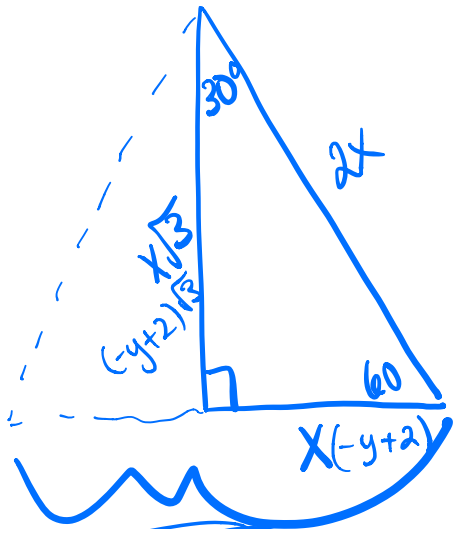
$$\left(\frac{f(x)}{2}\right)^2 + h^2 = f(x)^2$$

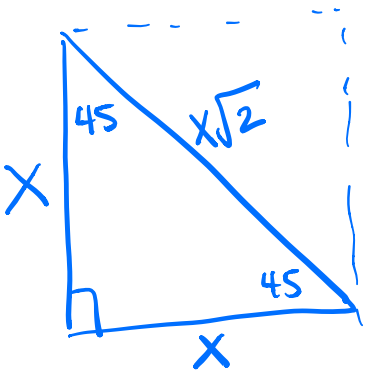
$$\frac{f(x)^2}{4} + h^2 = f(x)^2$$

$$h^2 = \frac{3}{4} f(x)^2$$

$$h = \frac{\sqrt{3}}{2} f(x)$$







$-2y + 4$