KEY

Discovering the Fundamental Theorem of Calculus

Objective: Given a function **y = f(t)**, find an "area function" **A(x)**, defined to be $A(x)=\int_a^x f(t)dt$ which represents the area "under" **f** from **t = a** to **t = x**. $a \to$ CONSTANT $x \to$ INPUT VARIABLE

Instructions: Complete steps **a** through **d** below for the given functions and intervals.

a. Make a sketch of the function **y = f(t)** and shade the region on the interval given.

b. Use a geometric formula to find the area of the shaded region. Your answer should be a function of x and, in fact, be $A(x)=\int_a^x f(t)dt$. If necessary, simplify your answer by multiplying the terms.

c. Check your answer (function) by graphing. Enter into Y1, $Y1=\int_a^x f(t)dt$, and into Y2, your answer for **A(x)**. If the two graphs are not the same curve, find your mistake and regraph.

d. Record your answer on this worksheet.

Examples: 1. f(t) = 5 on [0, x]. $A(x)=\int_0^x 5dt=$ $5x+0$

2. f(t) = t on [2 , x]. $A(x)=\int_2^x t\,dt=$ $\frac{1}{2}x^2-2$

Problems: 3. f(t) = 3 on [-1 , x .] $A(x)=\int_{-1}^x 3dt=$ $3x+3$

4. f(t) = t + 3 on [0 , x]. $A(x)=\int_0^x (t+3)dt=$ $\frac{1}{2}x^2+3x+0$

5. f(t) = 4t on [0 , x]. $A(x)=\int_0^x 4t\,dt=$ $2x^2$

$\int 5dx=5x+C$

6. f(t) = 4t on [1 , x]. $A(x)=\int_1^x 4t\,dt=$ $2x^2-2$

7. f(t) = 2t + 3 on [0 , x]. $A(x)=\int_0^x (2t+3)dt=$ x^2+3x+0

$\int x\,dx=\frac{1}{2}x^2+C$

8. f(t) = 2t + 3 on [-1 , x]. $A(x)=\int_{-1}^x (2t+3)dt=$ x^2+3x+2

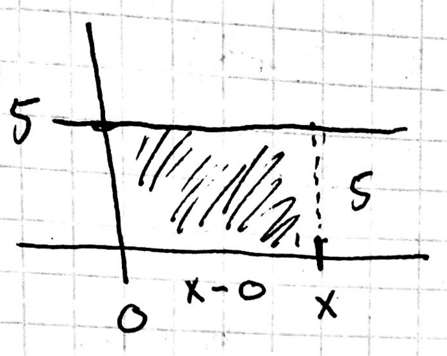
9. f(t) = -3t + 7 on [-2 , x] $A(x)=\int_{-2}^x (-3t+7)dt=$ $-\frac{3}{2}x^2+7x+20$

10. f(t) = -3t + 7 on [1 , x] $A(x)=\int_1^x (-3t+7)dt=$ $-\frac{3}{2}x^2+7x-\frac{11}{2}$

Conjecture: What is the relationship between the "area function" **A(x)** and the original function **f(t)**?

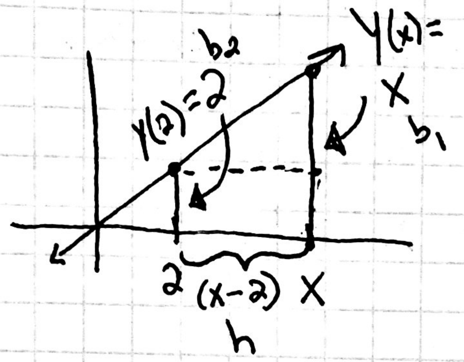
A(x) IS THE ANTIDERIVATIVE OF f(t)

① a)



b) $A(x) = 5\cdot(x-0) = 5x$

② $y = t$

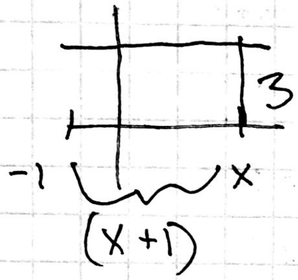


b) $A = \frac{1}{2}(b_1 + b_2)h$

$A(x) = \frac{1}{2}(x+2)(x-2)$

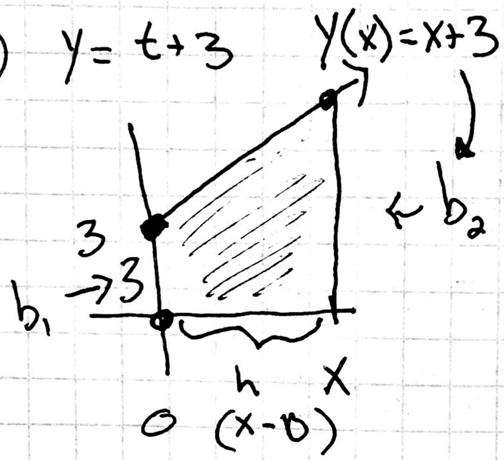
$A(x) = \frac{1}{2}(x^2-4) = \frac{1}{2}x^2 - 2$

③



$3x+3$

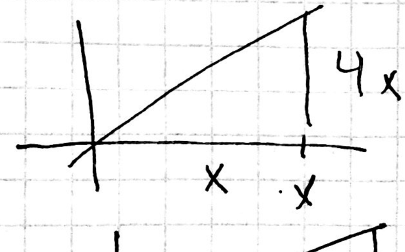
④ $y = t+3$



$A = \frac{1}{2}(x+3+3)\cdot x$

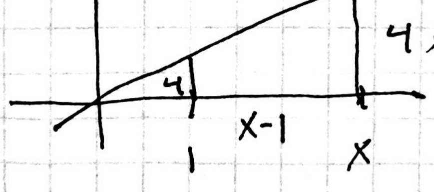
$= \frac{x^2}{2} + 3x$

⑤



$A(x) = \frac{1}{2}x(4x) = 2x^2$

⑥

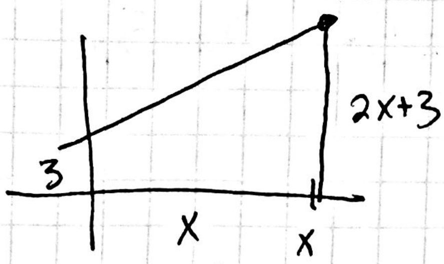


$A(x) = (x-1)\cdot\frac{1}{2}(4x+4)$

$= (x-1)(2x+2)$

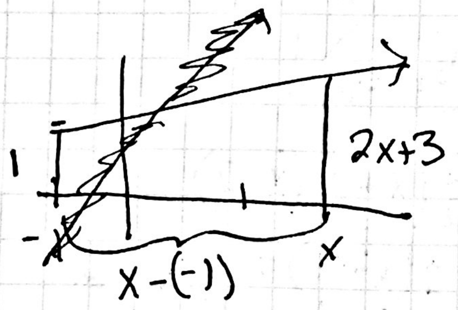
$= 2x^2 - 2$

(7) $y = 2t + 3$



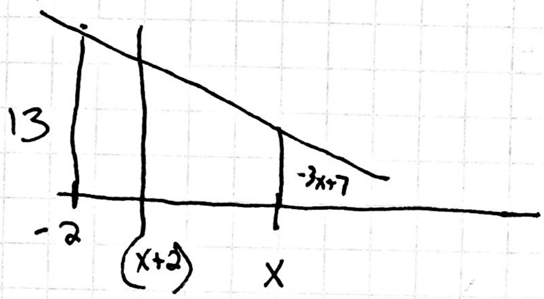
$$A(x) = \cancel{\tfrac{1}{2} \cdot 3 \cdot (2x+3)}$$
$$\frac{1}{2}x\,(2x+3+3)$$
$$= x^2 + 3x$$

(8)



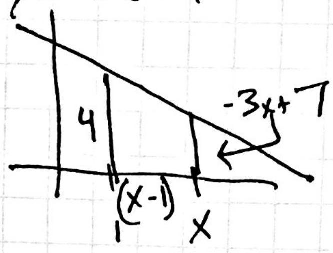
$$\begin{array}{ccc} & h & b_1 + b_2 \end{array}$$
$$A(x) = \frac{1}{2}(x+1)(2x+3+1)$$
$$= (x+1)(x+2)$$
$$= x^2 + 3x + 2$$

(9) $y = -3t + 7 \quad [-2, x]$



$$A(x) = \frac{1}{2}(x+2)(13 - 3x + 7)$$
$$= \frac{1}{2}(x+2)(-3x+20)$$
$$= \frac{1}{2}(-3x^2 + 14x + 40)$$
$$= -\frac{3}{2}x^2 + 7x + 20$$

(10) $y = -3t + 7$



$$A(x) = \frac{1}{2}(x-1)(-3x+7+4)$$
$$= \frac{1}{2}(x-1)(-3x+11)$$
$$= \frac{1}{2}(-3x^2 + 14x - 11)$$
$$= -\frac{3}{2}x^2 + 7x - \frac{11}{2}$$

LO 3.2A Interpret the definite integral as the limit of a Riemann sum. Express the limit of a Riemann sum in integral notation.

Instructions: Fill in the missing integral expression in the left column or the appropriate limit of a Riemann sum in the right column.

1. $\int_1^3 (4x^2 + 2)\, dx$ — $\lim_{n\to\infty} \sum_{j=1}^{n} \left[4\left(1+\frac{2}{n}j\right)+2\right]\cdot\frac{2}{n}$

2. $\int_2^5 (4x^2+2)\, dx$ — $\lim_{n\to\infty} \sum_{j=1}^{n} \left[4\left(2+\frac{3j}{n}\right)^2+2\right]\left(\frac{3}{n}\right)$

3. $\int_7^5 (3x + 1)\, dx$ — $\lim_{n\to\infty} \sum_{j=7}^{n} \left[3\left(7+\frac{-2}{n}j\right)+1\right]\cdot\frac{-2}{n}$

4. $\int_2^4 (4x+2)\, dx$ — $\lim_{n\to\infty} \sum_{j=1}^{n} \left[4\left(2+\frac{2j}{n}\right)^2+2\right]\left(\frac{2}{n}\right)$

5. $\int_5^2 (4x - 2)\, dx$ — $\lim_{n\to\infty} \sum_{j=1}^{n} \left[4\left(5+\frac{-3}{n}j\right)-2\right]\cdot\frac{-3}{n}$

6. $\int_3^1 (4x+2)\, dx$ — $\lim_{n\to\infty} \sum_{j=1}^{n} \left[4\left(3-\frac{2j}{n}\right)^2+2\right]\left(\frac{-2}{n}\right)$

7. $\int_5^7 (4x - 2)\, dx$ — $\lim_{n\to\infty} \sum_{j=1}^{n} \left[4\left(5+\frac{2}{n}j\right)-2\right]\cdot\frac{2}{n}$

8. $\int_2^5 3x+1\, dx$ — $\lim_{n\to\infty} \sum_{j=1}^{n} \left[3\left(2+\frac{3j}{n}\right)+1\right]\left(\frac{3}{n}\right)$

9. $\int_5^7 (x^3 + 1)\, dx$ — $\lim_{n\to\infty} \sum_{j=1}^{n} \left[\left(5+\frac{2}{n}j\right)^3+1\right]\cdot\frac{2}{n}$

10. $\int_2^5 x^3+1\, dx$ — $\lim_{n\to\infty} \sum_{j=1}^{n} \left[\left(2+\frac{3j}{n}\right)^3+1\right]\left(\frac{3}{n}\right)$

LO 3.2A Interpret the definite integral as the limit of a Riemann sum. Express the limit of a Riemann sum in integral notation.

Instructions: Match the integral expression in the left column with the appropriate limit of a Riemann sum in the right column.

$\Delta x = \frac{3-1}{n}$ $x_i = 1 + \frac{2}{n} i$ $f\left(1+\frac{2}{n}i\right) = \left[4\left(1+\frac{2}{n}i\right)^2 + 2\right]$

1. $\int_1^3 (4x^2 + 2)\, dx$ D

a. $\lim_{n\to\infty} \sum_{j=1}^{n} \left[4\left(5 - \frac{3j}{n}\right) - 2\right]\left(\frac{-3}{n}\right)$

$\Delta x = 3/n$, $a = 2$ 2. $\int_2^5 (x^3 + 1)\, dx$ C

b. $\lim_{n\to\infty} \sum_{j=1}^{n} \left[4\left(2 + \frac{3j}{n}\right)^2 + 2\right]\left(\frac{3}{n}\right)$

$\Delta x = \frac{-2}{n}$, $a = 7$ 3. $\int_7^5 (3x + 1)\, dx$ e

c. $\lim_{n\to\infty} \sum_{j=1}^{n} \left[\left(2 + \frac{3j}{n}\right)^3 + 1\right]\left(\frac{3}{n}\right)$

$\Delta x = \frac{2}{n}$, $a = 2$ 4. $\int_2^4 (4x^2 + 2)\, dx$ j

d. $\lim_{n\to\infty} \sum_{j=1}^{n} \left[4\left(1 + \frac{2j}{n}\right)^2 + 2\right]\left(\frac{2}{n}\right)$

$\Delta x = -3/n$, $a = 5$ 5. $\int_5^2 (4x - 2)\, dx$ a

e. $\lim_{n\to\infty} \sum_{j=1}^{n} \left[3\left(7 - \frac{2j}{n}\right) + 1\right]\left(\frac{-2}{n}\right)$

$\Delta x = 3/n$, $a = 2$ 6. $\int_2^5 (4x^2 + 2)\, dx$ b

f. $\lim_{n\to\infty} \sum_{j=1}^{n} \left[4\left(3 - \frac{2j}{n}\right)^2 + 2\right]\left(\frac{-2}{n}\right)$

$\Delta x = \frac{2}{n}$, $a = 5$ 7. $\int_5^7 (4x - 2)\, dx$ h

g. $\lim_{n\to\infty} \sum_{j=1}^{n} \left[3\left(2 + \frac{3j}{n}\right) + 1\right]\left(\frac{3}{n}\right)$

$\Delta x = \frac{-2}{n}$, $a = 3$ 8. $\int_3^1 (4x^2 + 2)\, dx$ f

h. $\lim_{n\to\infty} \sum_{j=1}^{n} \left[4\left(5 + \frac{2j}{n}\right) - 2\right]\left(\frac{2}{n}\right)$

$\Delta x = \frac{2}{n}$, $a = 5$ 9. $\int_5^7 (x^3 + 1)\, dx$ i

i. $\lim_{n\to\infty} \sum_{j=1}^{n} \left[\left(5 + \frac{2j}{n}\right)^3 + 1\right]\left(\frac{2}{n}\right)$

10. $\int_2^5 (3x + 1)\, dx$ g

j. $\lim_{n\to\infty} \sum_{j=1}^{n} \left[4\left(2 + \frac{2j}{n}\right)^2 + 2\right]\left(\frac{2}{n}\right)$