Calculus   Name____________________ ID: 1

Create the ANSWER KEY   Date____________ Period____

Evaluate each definite integral.

1) $\int_{-4}^{-2} (-x^2 - 4x - 3)\, dx$

$-\frac{2}{3} \approx -0.667$

2) $\int_{-\frac{\pi}{6}}^{0} \sec x \tan x\, dx$

$\frac{3 - 2\sqrt{3}}{3} \approx -0.155$

3) $\int_{-1}^{3} 3(2x + 2)^{\frac{1}{3}}\, dx$

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4) $\int_{1}^{2} 2e^{2x - 4}\, dx$

$\frac{e^2 - 1}{e^2} \approx 0.865$

5) $\int_{0}^{1} -\frac{4x}{(x^2 + 1)^2}\, dx$

-1

6) $\int_{-4}^{1} -|x^2 + 3x|\, dx$

$-\frac{49}{6} \approx -8.167$

7) $\int_{-\frac{\pi}{4}}^{-\frac{\pi}{6}} -2\sec^2 x\, dx$

$\frac{-6 + 2\sqrt{3}}{3} \approx -0.845$

8) $\int_{-1}^{1} -3x^2(x^3 + 1)^2\, dx$

$-\frac{8}{3} \approx -2.667$

For each problem, find $F'(x)$.

9) $F(x) = \int_{-4}^{x} \frac{3}{t}\, dt$

$F'(x) = \frac{3}{x}$

10) $F(x) = \int_{0}^{x^3} (t - 1)\, dt$

$F'(x) = 3x^5 - 3x^2$

11) $F(x) = \int_{2x}^{1} (-t^3 + t^2 - 1)\, dt$

$F'(x) = 16x^3 - 8x^2 + 2$

12) $F(x) = \int_{x}^{x^2} 4t^{\frac{1}{3}}\, dt$

$F'(x) = 8x^{\frac{5}{3}} - 4x^{\frac{1}{3}}$

1

$$\int_{-4}^{-2} (-x^2 - 4x - 3)\,dx$$

$F(-2) - F(-4)$

$\frac{2}{3} - \frac{4}{3}$

$= \boxed{-\frac{2}{3}}$

$F(x) = -\frac{x^3}{3} - \frac{4x^2}{2} - 3x$

$-\frac{1}{3}x^3 - 2x^2 - 3x$

$F(-2) = -\frac{1}{3}(-2)^3 - 2(-2)^2 - 3(-2)$

$-\frac{1}{3}(-8) - 2(4) + 6$

$\frac{8}{3} - 8 + 6$

$\frac{8}{3} - 2 \rightarrow \frac{8}{3} - \frac{6}{3} = \frac{2}{3}$

$F(-4) = -\frac{1}{3}(-4)^3 - 2(-4)^2 - 3(-4)$

$-\frac{1}{3}(-64) - 2(16) + 12$

$\frac{64}{3} - 32 + 12$

$\frac{64}{3} - 20 \rightarrow \frac{64}{3} - \frac{60}{3} = \frac{4}{3}$

#1) $\int 2e^{2x-4}\,dx$

Adam Paul

Tarra

answer key #2

$$\int_{-\frac{\pi}{6}}^{0} \sec x \tan x \, dx \qquad F(x) = \sec x$$

$$F(0) - F\left(-\frac{\pi}{6}\right) = \int_{-\frac{\pi}{6}}^{0} \sec x \tan x \, dx$$

$$\downarrow \qquad \downarrow$$

$$\sec(0) - \sec\left(-\frac{\pi}{6}\right) =$$

$$\downarrow$$

$$1 - \frac{2}{\sqrt{3}} = \int_{-\frac{\pi}{6}}^{0} \sec x \tan x \, dx$$

$$\downarrow \qquad \downarrow$$

$$\frac{3}{3} \qquad \frac{-2}{\sqrt{3}} \rightarrow \frac{-2\sqrt{3}}{3}$$

$$\boxed{\frac{3 - 2\sqrt{3}}{3}} = \int_{-\frac{\pi}{6}}^{0} \sec x \tan x \, dx$$

4) $\int^{12} 7^{2x-4}\,dx$

Ethan, Kellie, Zack

Problem #3

$\int_{-1}^{3} \left(3(2x+2)^{1/3}\right) dx$

$u = 2x+2$ $\quad$ $2(-1)+2 = 0$

$du = 2dx$ $\quad$ $2(3)+2 = 8$

$\frac{du}{2} = dx$

$3\int_{-1}^{3} (u)^{1/3} \cdot \frac{du}{2}$

$3 \cdot \frac{1}{2} \int_{0}^{8} u^{1/3} \cdot du$

$\frac{3}{2} \cdot \left[\frac{u^{4/3}}{4/3}\right]$

$\frac{3}{2} \cdot \frac{3}{4} (u)^{4/3}$

$\frac{9}{8} (u)^{4/3}$

$F(0) = \frac{9}{8}(0)^{4/3} = 0$

$F(8) = \frac{9}{8}(8)^{4/3} = \frac{9}{8}(16) = 18$

$F(8) - F(0) = 18 - 0 = \boxed{18}$

4) $\int_1^2 2e^{2x-4}\,dx$

$u = 2x-4$

$du = 2\,dx$

$$\int_1^2 \cancel{2}e^{2x-4}\,\cancel{dx}$$

$$\int_1^2 e^u\,du$$

$$e^{2x-4} \rightarrow 1-e^{-2}$$

$$\boxed{\frac{e^2-1}{e^2}}$$

$$1-\frac{1}{e^2} \rightarrow \frac{e^2}{e^2}-\frac{1}{e^2} \rightarrow \frac{e^2-1}{e^2}$$

(5) $\int_0^1 \frac{-4x}{(x^2+1)^2}\,dx$

$u = x^2+1$

$du = 2x\,dx$

$\frac{du}{2} = x\,dx$

$= -4\left(\frac{1}{2}\right)\int_0^1 \frac{du}{u^2}$

$f(0) = \frac{2}{0^2+1}$

$= -2\int_0^1 (du)(u^{-2})$

$f(0) = 2$

$f(1) = \frac{2}{1^2+1} = \frac{2}{2}$

$= -2\left[\frac{u^{-1}}{-1}\right]$

$f(1) = 1$

$= 2\left[x^2+1\right]^{-1}$

$f(1) - f(0)$

$1 - 2$

$= \frac{2}{x^2+1}$

$\boxed{= -1}$

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Bethany, Raquel
Lily

answer key # 6

$$\int_{-4}^{1} -|x^2+3x|\,dx \qquad \frac{-49}{6}$$

$$= -\int_{-4}^{1} |x^2+3x|\,dx$$

$x^2+3x=0$

$x(x+3)=0$

$x=0,-3$

$$-\left[\int_{-4}^{-3} x^2+3x\,dx - \int_{-3}^{0} x^2+3x\,dx + \int_{0}^{1} x^2+3x\,dx\right]$$

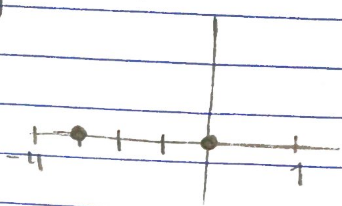
$+$ $(-)(-)$ -4 -3 $-$ $(-)(+)$ 0 $+$ $(+)(+)$ 1

split into 3 integrals

$$\int_{-4}^{-3} x^2+3x\,dx = F(-3)-F(-4)$$

$$\int_{-3}^{0} x^2+3x\,dx = F(0)-F(-3)$$

$$\int_{0}^{1} x^2+3x\,dx = F(1)-F(0)$$



$$F(x) = \frac{1}{3}x^3 + \frac{3}{2}x^2$$

$$F(-4) = \frac{1}{3}(-4)^3 + \frac{3}{2}(-4)^2$$

$$= \frac{-64}{3} + \frac{3\cdot 8\cdot \cancel{2}}{\cancel{2}} = \frac{-64}{3} + \frac{24\cdot 3}{3} = \frac{8}{3}$$

$$F(-3) = \frac{1}{3}(-3)^3 + \frac{3}{2}(3)^2$$

$$= \frac{-3\cdot 3\cdot \cancel{3}}{\cancel{3}} + \frac{3\cdot 3\cdot 3}{2} = \frac{-2\cdot 9}{2} + \frac{27}{2} = \frac{9}{2}$$

$$F(0) = 0$$

$$F(1) = \frac{2}{6}\cancel{\frac{1}{3}} + \frac{9}{6}\cancel{\frac{3}{2}} = \frac{11}{6}$$

$$-\left[\left(F(-3)-F(-4)\right) - \left(F(0)-F(-3)\right) + \left(F(1)-F(0)\right)\right]$$

$$-\left[\left(\frac{27}{6}-\frac{16}{6}\right) - \left(-\frac{27}{6}\right) + \left(\frac{11}{6}\right)\right]$$

$$-\left[\frac{11}{6} + \frac{27}{6} + \frac{11}{6}\right]$$

$$\frac{9}{2} = \frac{27}{6} \qquad \frac{8}{3} = \frac{16}{6}$$

$$\frac{-11}{6} - \frac{27}{6} - \frac{11}{6} = \boxed{\frac{-49}{6}}$$

8)

Will, Nathan, Damon

Problem 7

$\int_{-\pi/4}^{-\pi/6} -2\sec^2 x \, dx$ $\frac{d}{dx}\tan x = \sec^2 x$

$\int_{-\pi/4}^{-\pi/6} -2\tan x \, dx$

$\tan \pi/6 = \frac{1/2}{\sqrt{3}/2} = \frac{1}{\sqrt{3}}$ $\tan -\pi/6 = \frac{-1}{\sqrt{3}}$

$\tan \pi/4 = 1$ $\tan -\pi/4 = -1$

$-2[\tan(-\pi/4) + \tan(-\pi/6)]$

$= -2[-1 + -\frac{1}{\sqrt{3}}]$

$\therefore \boxed{\frac{2}{\sqrt{3}} - 2}$

8) $\int_{-1}^{1} -3x^2 \underbrace{(x^3+1)}_{u}^2 \, dx$

$u = x^3+1$

$du = 3x^2 \, dx$

$-1 \int_{-1}^{1} du (u)^2$

$-1 \left(\frac{u^3}{3} \right) \Big|_0^2 \rightarrow \boxed{-\frac{8}{3}}$

Raquel, Bethany,
Adam Paul
Tavra

answer key #9

$$F(x) = \int_{-4}^{x} \frac{3}{t}\,dt$$

$$F'(x) = \frac{d}{dx}\left[\int_{-4}^{x} \frac{3}{t}\,dt\right]$$

$$\frac{3}{x} \cdot \underset{\substack{\uparrow \\ \frac{d}{dx} \text{ of } (x)}}{1} = \frac{3}{x}$$

$$\boxed{F'(x) = \frac{3}{x}}$$

Raquel, Bethany,
Lilu

Problem #10

10) $F(x) = \int_0^{x^3} (t-1)\,dt$

$\frac{d}{dx}\left[\int_{\cancel{0}}^{x^3} (t-1)\,dt\right]$

$(x^3-1)\cdot 3x^2$

$3x^5 - 3x^2$

Raquel, Bethany, Lily

answerkey # 11

$$F(x) = \int_{x=2x}^{x=1} (-t^3+t^2-1)\,dt \qquad F'(x) = 16x^3 - 8x^2 + 2$$

$$= -\int_{1}^{2x} (-t^3+t^2-1)\,dt$$

$$= -\left[-(2x)^3 + (2x)^2 - 1\right] \cdot \frac{d}{dx}\left[2x\right]$$

$$= -(-8x^3+4x^2-1)\,2$$

$$= \boxed{+16x^3 - 8x^2 + 2} \checkmark$$

#12 $F(x) = \int_{x}^{x^2} 4t^{1/3}\,dt$

$$F(x) = \int_{x}^{a} 4t^{1/3}\,dt + \int_{a}^{x^2} 4t^{1/3}\,dt$$

$$= -\int_{a}^{x} 4t^{1/3}\,dt + \int_{a}^{x^2} 4t^{1/3}\,dt$$

$$F'(x) = \left[-4(x)^{1/3}\right] + \left[4(x^2)^{1/3} \cdot 2x\right]$$

$$= -4x^{1/3} + \left[4x^{2/3} \cdot 2x\right]$$

$$\boxed{F'(x) = -4x^{1/3} + 8x^{5/3}}$$