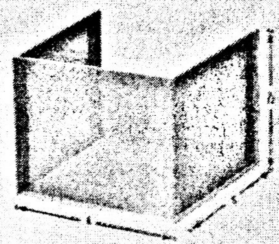
Date: 12/2/09

Lesson Title: 3.7 Optimization part 2

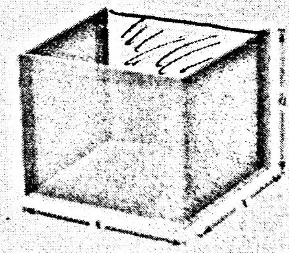
Objective: To solve real-life optimization problems.

IN: A manufacturer wants to design an open box having a square base and a surface area of 108 square inches. **Find the dimensions of three different boxes that satisfy the criteria.**



Open box with square base.
$S = x^2 + 4xh = 108$

A manufacturer wants to design an open box having a square base and a surface area of 108 square inches. **What dimensions will produce a box with maximum volume?**



Open box with square base.
$S = x^2 + 4xh = 108$

$V = x^2 \cdot h$

$SA = x^2 + 4xh$

$108 = x^2 + 4xh$

$\frac{108 - x^2}{4x} = h$

$V = x^2\left(\frac{108 - x^2}{4x}\right)$

$V = \frac{1}{4}(108x - x^3)$

$V' = \frac{1}{4}(108) - \frac{1}{4}(3x^2)$

$0 = 108 - 3x^2$

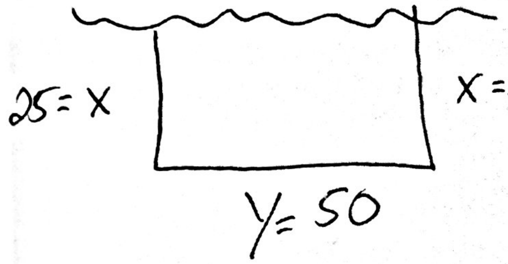
$3x^2 = 108$

$x^2 = 36$

$x = \pm 6$

$\boxed{6 \times 6 \times 3}$

A farmer has 100 feet of fence and wants to make a rectangular pigpen, one side of which is along an existing straight fence. What dimensions should be used in order to maximize the area of the pen?



$2x+y=100$

$M=xy$

$M=x(100-2x)$

$M=100x-2x^2$

$M'=100-4x$

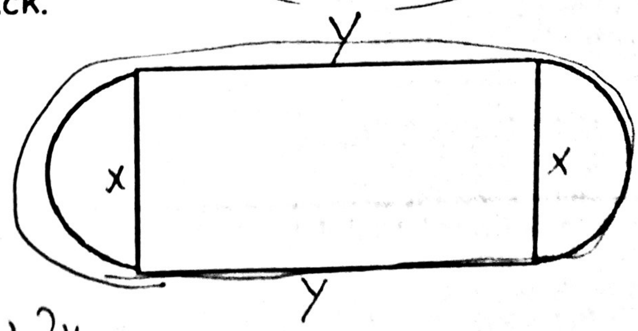
$0=100-4x$

$4x=100$

$\boxed{x=25}$

$\boxed{\text{25 BY 50 FEET}}$

A university is building a new running track. It is to be the perimeter of a region obtained by putting two semi-circles on the ends of a rectangle as shown below. If the track is to be 440 yards long, determine the necessary dimensions to build the track in order to maximize the area surrounded by the track.



$2\pi\left(\frac{x}{2}\right)^2+2y$

$\frac{2\pi x^2}{4}+2y$

$\frac{1}{2}\pi x^2+2y=440$

$2y=440-\frac{1}{2}\pi x^2$

$y=220-\frac{\pi}{4}x^2$

$M=x\cdot y+\pi\left(\frac{x}{2}\right)^2$

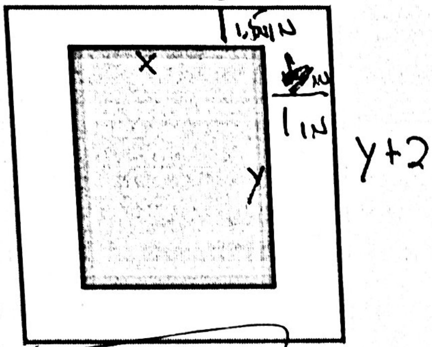
$M=x\left(220-\frac{\pi}{4}x^2\right)+\frac{\pi}{4}x^2$

$M=220x-\frac{\pi}{4}x^3+\frac{\pi}{4}x^2$

$0=220-\frac{3\pi}{4}x^2+\frac{\pi}{2}x$

$x=\cancel{9.335}$ 10.002

A rectangular page is to contain 24 square inches of print. The margins at the top and bottom of the page are to be $1\frac{1}{2}$ inches, and the margins on the left and right are to be 1 inch. What should the dimensions of the page be so that the least amount of paper is used?



$\boxed{9'' \times 6''}$

PRIM

$M = (x+3)(y+2)$

$M = (x+3)\left(\frac{24}{x}+2\right)$

$M = 24 + 72x^{-1} + 2x + 6$

$M' = -72x^{-2} + 2$

$-2 = \frac{-72}{x^2}$

$-2x^2 = -72$

$x^2 = 36$

$x = \pm 6$

SEC

$xy = 24$

$y = \frac{24}{x}$

$x = 6,\ y = 4$

Four feet of wire is to be used to form a square and a circle. How much of the wire should be used for the square and how much should be used for the circle to enclose the maximum total area?

PRIMARY

$M = x^2 + \pi y^2$

$M = x^2 + \pi\left(\frac{2-2x}{\pi}\right)^2$

$M = x^2 + \pi \frac{(2-2x)^2}{\pi^2}$

$M = x^2 + \frac{(2-2x)^2}{\pi}$

~~$0 = \pi x^2 + (2-2x)^2$~~

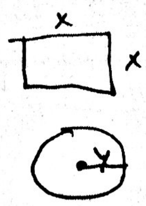
$M' = 2x + \frac{1}{\pi}2(2-2x)\cdot(-2)$

$0 = 2x - \frac{4}{\pi}(2-2x)$

$0 = 2x - \frac{8}{\pi} + \frac{8}{\pi}x \longrightarrow \frac{8}{\pi} = \left(2+\frac{8}{\pi}\right)x$

SECONDARY

$4 = 4x + 2\pi y$



$\frac{2\pi y}{2\pi} = \frac{4-4x}{2\pi}$

$y = \frac{2-2x}{\pi}$

SQUARE $\boxed{2.24 \text{ ft}}$

CIRCLE $\boxed{1.760 \text{ ft}}$

$\boxed{x \approx .56}$ $\boxed{y \approx .28}$

Out:

Which points on the graph of $y = 4 - x^2$ are closest to the point (0,2)?

$(x, 4-x^2)$

$(0,2)$

$M^2 = (x-0)^2 + ((4-x^2)-2)^2$

$M^2 = x^2 + (2-x^2)^2$

$M^2 = x^2 + 4 - 4x^2 + x^4$

$\sqrt{M^2} = \sqrt{x^4 - 3x^2 + 4}$

$M = (x^4 - 3x^2 + 4)^{1/2}$

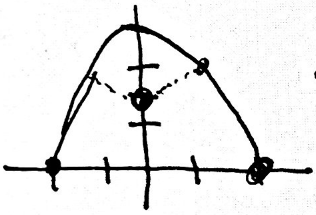
$M' = \frac{1}{2}(x^4 - 3x^2 + 4)^{-1/2} \cdot (4x^3 - 6x)$

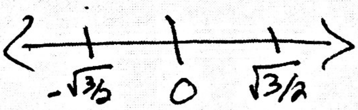
$0 = \frac{(4x^3 - 6x)}{2\sqrt{x^4 - 3x^2 + 4}} = \frac{2x(2x^2-3)}{2\sqrt{\ }}$

Summary:

I wonder...

Pic

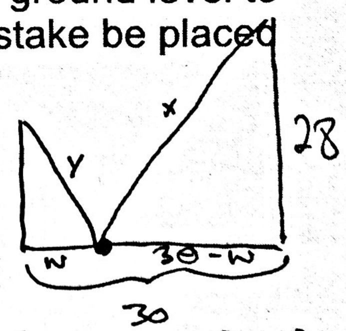




$x = 0, \sqrt{\frac{3}{2}}, -\sqrt{\frac{3}{2}}$

Finding Minimum Length

Two posts, one 12 feet high and the other 28 feet high, stand 30 feet apart. They are to be stayed by two wires, attached to a single stake, running from ground level to the top of each post. Where should the stake be placed to use the least wire?



$M = x + y$

$M = \sqrt{144+w} + \sqrt{1684-60w+w^2}$

$M = (144+w)^{1/2} + (1684-60w+w^2)^{1/2}$

$w^2 + 12^2 = y^2$

$(30-w)^2 + 28^2 = x^2$

$900 - 60w + w^2$

$1684 - 60w + w^2$

$M' = \frac{1}{2}(144+w)^{-1/2} + \frac{1}{2}(1684-60w+w^2)^{-1/2} \cdot (-60+2w)$

$0 = \frac{1}{2(144+w)^{1/2}} + \frac{2w-60}{2(w^2-60w+1684)^{1/2}}$

$\boxed{\text{28.935 ft AWAY FROM THE 12 ft POST}}$