3. Evaluate each limit using algebraic techniques.
Use ∞, $-\infty$ or DNE where appropriate.

(a) $\lim_{x\to 0} \dfrac{x^2-25}{x^2-4x-5}$

(b) $\lim_{x\to 5} \dfrac{x^2-25}{x^2-4x-5}$

(c) $\lim_{x\to 1} \dfrac{7x^2-4x-3}{3x^2-4x+1}$

(d) $\lim_{x\to -2} \dfrac{x^4+5x^3+6x^2}{x^2(x+1)-4(x+1)}$

(e) $\lim_{x\to -3} |x+1| + \dfrac{3}{x}$

(f) $\lim_{x\to 3} \dfrac{\sqrt{x+1}-2}{x^2-9}$

(g) $\lim_{x\to 3} \dfrac{\sqrt{x^2+7}-3}{x+3}$

(h) $\lim_{x\to 2} \dfrac{x^2+2x-8}{\sqrt{x^2+5}-(x+1)}$

(i) $\lim_{y\to 5} \left(\dfrac{2y^2+2y+4}{6y-3}\right)^{1/3}$

(j) $\lim_{x\to 0} \sqrt[4]{2\cos(x)-5}$

(k) $\lim_{x\to 0} \dfrac{\dfrac{1}{3+x}-\dfrac{1}{3-x}}{x}$

(l) $\lim_{x\to -6} \dfrac{\dfrac{2x+8}{x^2-12}-\dfrac{1}{x}}{x+6}$

(m) $\lim_{x\to \infty} \sqrt{x^2-2}-\sqrt{x^2+1}$

(n) $\lim_{x\to -\infty} \sqrt{x-2}-\sqrt{x}$

(o) $\lim_{x\to 7} \sqrt[6]{2x-14}$

(p) $\lim_{x\to 1^-} \sqrt{3-3x}$

(q) $\lim_{x\to \infty} \dfrac{x^4-10}{4x^3+x}$

(r) $\lim_{x\to -\infty} \sqrt[3]{\dfrac{x-3}{5-x}}$

(s) $\lim_{x\to \infty} \dfrac{3x^3+x^2-2}{x^2+x-2x^3+1}$

(t) $\lim_{x\to \infty} \dfrac{x+5}{2x^2+1}$

(u) $\lim_{x\to -\infty} \cos\left(\dfrac{x^5+1}{x^6+x^5+100}\right)$

(v) $\lim_{x\to 2} \dfrac{2x}{x^2-4}$

(w) $\lim_{x\to -1} \dfrac{3x}{x^2+2x+1}$

(x) $\lim_{x\to -1} \dfrac{x^2-25}{x^2-4x-5}$

(y) $\lim_{x\to 3} \dfrac{\sqrt{x^2-5}+2}{x-3}$

(z) $\lim_{x\to 0} \dfrac{2^x+\sin(x)}{x^4}$

(A) $\lim_{x\to 1^-} \dfrac{1}{x-1}+e^{x^2}$

(B) $\lim_{x\to \infty} 2x^2-3x$

(C) $\lim_{x\to 0} \dfrac{\sqrt{x+2}-\sqrt{2-x}}{x}$

(D) $\lim_{x\to 0^+} \dfrac{e^x}{1+\ln(x)}$

(E) $\lim_{x\to \infty} \sqrt{x^2+1}-2x$

(F) $\lim_{x\to 1} \dfrac{\sqrt[3]{x}-1}{\sqrt{x}-1}$

(b) $\lim_{x \to 5} \dfrac{x^2-25}{x^2-4x-5} = \lim_{x \to 5} \dfrac{(x+5)\cancel{(x-5)}}{\cancel{(x-5)}(x+1)} = \dfrac{5+5}{5+1} = \boxed{\dfrac{5}{3}}$

(i) $\lim_{x \to 5} \left(\dfrac{2y^2+2y+4}{6y-3}\right)^{1/3} = \left(\dfrac{2(5)^2+2(5)+4}{6(5)-3}\right)^{1/3} = \left(\dfrac{50+10+4}{30-3}\right)^{1/3}$

$= \left(\dfrac{64}{27}\right)^{1/3} = \left(\dfrac{4^3}{3^3}\right)^{1/3} = \boxed{\dfrac{4}{3}}$

(k) $\lim_{x \to 0} \dfrac{\frac{1}{3+x} - \frac{1}{3-x}}{x} = \lim_{x \to 0} \dfrac{\frac{3-x}{3-x} \cdot \frac{1}{3+x} - \frac{1}{3-x} \cdot \frac{3+x}{3+x}}{x}$

$= \lim_{x \to 0} \dfrac{\frac{(3-x)-(3+x)}{(3-x)(3+x)}}{x} = \lim_{x \to 0} \dfrac{-2x}{(3-x)(3+x)} \cdot \dfrac{1}{x} = \lim_{x \to 0} \dfrac{-2}{(3-x)(3+x)}$

$= \dfrac{-2}{(3)(3)} = \boxed{-2/9}$

(m) $\lim_{x \to \infty} \sqrt{\underbrace{x^2}_{\text{MUSC. TERM}}-2} - \sqrt{\underbrace{x^2}_{\text{MUSC. TERM}}+1} = \lim_{x \to \infty} \sqrt{x^2} - \sqrt{x^2}$

$= \lim_{x \to \infty} x - x = \lim_{x \to \infty} 0 = \boxed{0}$

(s) $\lim_{x \to \infty} \dfrac{\overset{\text{MUSC.}}{3x^3}+x^2-2}{x^2+x\underset{\text{MUSC.}}{-2x^3}+1} = \lim_{x \to \infty} \dfrac{3x^3}{-2x^3} = \lim_{x \to \infty} \dfrac{3}{-2} = \boxed{-\dfrac{3}{2}}$

(V) $\lim_{x\to 2} \frac{2x}{x^2-4} = \lim_{x\to 2} \frac{2x}{(x+2)(x-2)} = \boxed{\text{DNE (ASYMPTOTE)}}$

(X) $\lim_{x\to -1} \frac{x^2-25}{x^2-4x-5} = \lim_{x\to -1} \frac{(x+5)(x-5)}{(x-5)(x+1)} = \boxed{\text{DNE (ASYMPTOTE)}}$

(Y) $\lim_{x\to 3} \frac{\sqrt{x^2-5}+2}{x-3} \cdot \frac{\sqrt{x^2-5}-2}{\sqrt{x^2-5}-2} = \lim_{x\to 3} \frac{x^2-9}{(x-3)(\sqrt{x^2-5}-2)} = \lim_{x\to 3} \frac{(x+3)(x-3)}{(x-3)(\sqrt{x^2-5}-2)}$

$= \lim_{x\to 3} \frac{x+3}{\sqrt{x^2-5}-2} = \frac{3+3}{\sqrt{3^2-5}-2} = \frac{9}{\sqrt{4}-2} = \boxed{\text{DNE}}$

(B) $\lim_{x\to\infty} \underbrace{2x^2}_{\text{MUSCLE TERM}} - 3x = \lim_{x\to\infty} 2x^2 = \boxed{+\infty}$