



***Properties of Definite Integrals***

1. Given  $\int_0^5 f(x) dx = 10$  and  $\int_5^7 f(x) dx = 3$ , find

a)  $\int_0^7 f(x) dx$

b)  $\int_5^0 f(x) dx$

c)  $\int_5^5 f(x) dx$

d)  $\int_0^5 3f(x) dx$

2. Given  $\int_2^6 f(x) dx = 10$  and  $\int_2^6 g(x) dx = -2$ , find

a)  $\int_2^6 [f(x) + g(x)] dx$

b)  $\int_2^6 [g(x) - f(x)] dx$

c)  $\int_2^6 2g(x) dx$

d)  $\int_2^6 [2f(x) - 3g(x)] dx$

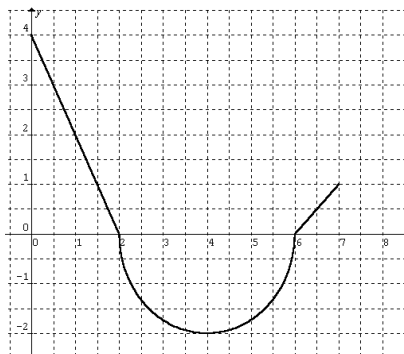
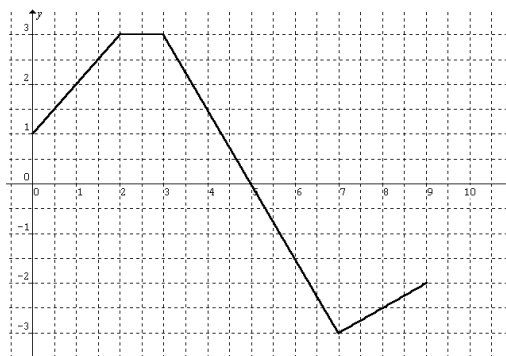
3. The graph of  $f(x)$  is shown. Evaluate each integral by interpreting it in terms of areas.

a)  $\int_0^2 f(x) dx$

b)  $\int_0^5 f(x) dx$

c)  $\int_5^7 f(x) dx$

d)  $\int_0^9 f(x) dx$



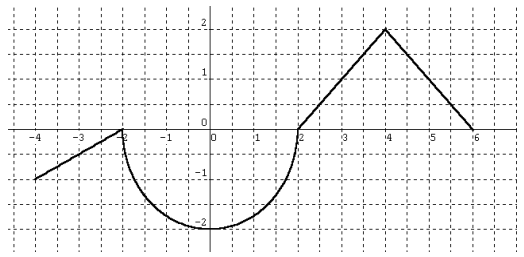
4. The graph of  $g(x)$  is shown. Evaluate each integral by interpreting it in terms of areas.

a)  $\int_0^2 g(x) dx$

b)  $\int_2^6 g(x) dx$

c)  $\int_0^7 g(x) dx$

5. The graph of  $f(x)$  is shown. Evaluate each integral by using geometric formulas.



a)  $\int_0^2 f(x) dx$                       b)  $\int_2^6 f(x) dx$   
c)  $\int_{-4}^2 f(x) dx$                       d)  $\int_{-4}^6 f(x) dx$   
e)  $\int_{-4}^2 |f(x)| dx$                       f)  $\int_{-4}^2 [f(x)+2] dx$

6. Consider the function  $f$  that is continuous in the interval  $[-5, 5]$  and for which  $\int_0^5 f(x) dx = 4$ .

Evaluate each integral.

a)  $\int_0^5 [f(x)+3] dx$                       b)  $\int_{-2}^3 f(x+2) dx$     (*Hint: assume the graph for  $f(x)$  is known, and sketch the graph of  $f(x+2)$* )  
c)  $\int_{-5}^5 f(x) dx$  ( $f$  is even.)                      d)  $\int_{-5}^5 f(x) dx$  ( $f$  is odd.)

In 7–10, determine whether the statement is true or false. If it is false, explain why or give an example that shows it is false.

7.  $\int_a^b [f(x)+g(x)] dx = \left[ \int_a^b f(x) dx \right] + \left[ \int_a^b g(x) dx \right]$

8.  $\int_a^b [f(x) \cdot g(x)] dx = \left[ \int_a^b f(x) dx \right] \cdot \left[ \int_a^b g(x) dx \right]$

9. The value of  $\int_a^b f(x) dx$  must be positive.

10. If  $\int_a^b f(x) dx > 0$ , then  $f$  is nonnegative for all  $x$  in  $[a, b]$ .

11. Evaluate, if possible, the integral  $\int_0^2 \lfloor x \rfloor dx$  (*Hint: sketch the graph of  $y = \lfloor x \rfloor$  for  $0 \leq x \leq 2$  first.*

Remember that  $y = \lfloor x \rfloor$  is the greatest integer function and it always rounds down to the nearest integer value.)

12. Sketch the region whose area is given by the definite integral. Then use geometric formulas to evaluate the integral.

a)  $\int_{-2}^2 (1-|x|) dx$                       b)  $\int_0^3 |3x-6| dx$



**Properties of Definite Integrals**

1. a)  $\int_0^7 f(x) dx = 13$

c)  $\int_5^5 f(x) dx = 0$

b)  $\int_5^0 f(x) dx = -10$

d)  $\int_0^5 3f(x) dx = 30$

2. a)  $\int_2^6 [f(x) + g(x)] dx = 8$

c)  $\int_2^6 2g(x) dx = -4$

b)  $\int_2^6 [g(x) - f(x)] dx = -12$

d)  $\int_2^6 [2f(x) - 3g(x)] dx = 26$

3. a)  $\int_0^2 f(x) dx = 4$

c)  $\int_5^7 f(x) dx = -3$

b)  $\int_0^5 f(x) dx = 10$

d)  $\int_0^9 f(x) dx = 2$

4. a)  $\int_0^2 g(x) dx = 4$

c)  $\int_0^7 g(x) dx = 4.5 - 2\pi \approx -1.783$

b)  $\int_2^6 g(x) dx = -2\pi$

5. a)  $\int_0^2 f(x) dx = -\pi$

c)  $\int_{-4}^2 f(x) dx = -2\pi - 1$

e)  $\int_{-4}^2 |f(x)| dx = 2\pi + 1$

b)  $\int_2^6 f(x) dx = 4$

d)  $\int_{-4}^6 f(x) dx = 3 - 2\pi$

f)  $\int_{-4}^2 [f(x) + 2] dx = 11 - 2\pi$

6. a)  $\int_0^5 [f(x) + 3] dx = 19$

c)  $\int_{-5}^5 f(x) dx = 8$  ( $f$  is even.)

b)  $\int_{-2}^3 f(x+2) dx = 4$

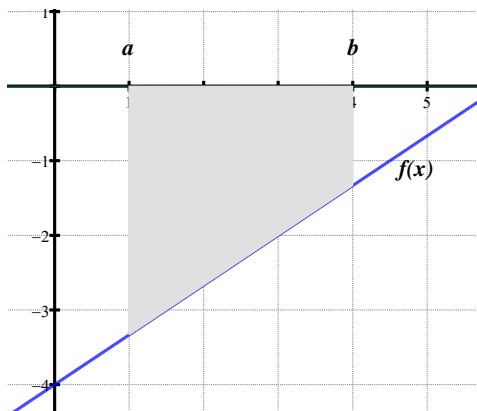
d)  $\int_{-5}^5 f(x) dx = 0$  ( $f$  is odd.)

7.  $\int_a^b [f(x) + g(x)] dx = \left[ \int_a^b f(x) dx \right] + \left[ \int_a^b g(x) dx \right]$  TRUE

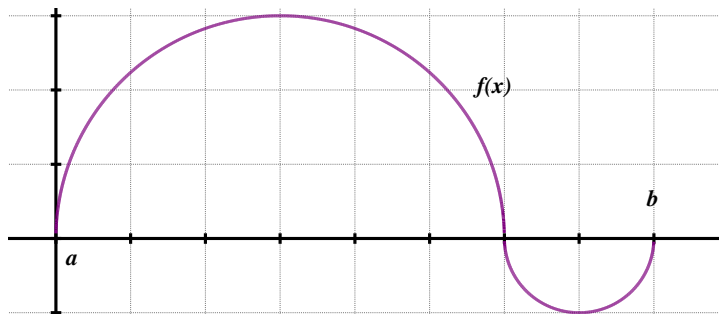
8.  $\int_a^b [f(x) \cdot g(x)] dx = \left[ \int_a^b f(x) dx \right] \cdot \left[ \int_a^b g(x) dx \right]$  FALSE: let  $f(x) = x$ ;  $g(x) = x$ ;  $a = 0$ ;  $b = 1$ .

We will have:  $\int_0^1 [x \cdot x] dx = \frac{1}{3} \neq \left[ \int_0^1 x dx \right] \cdot \left[ \int_0^1 x dx \right] = \frac{1}{4}$

9. The value of  $\int_a^b f(x) dx$  must be positive. FALSE (see below)

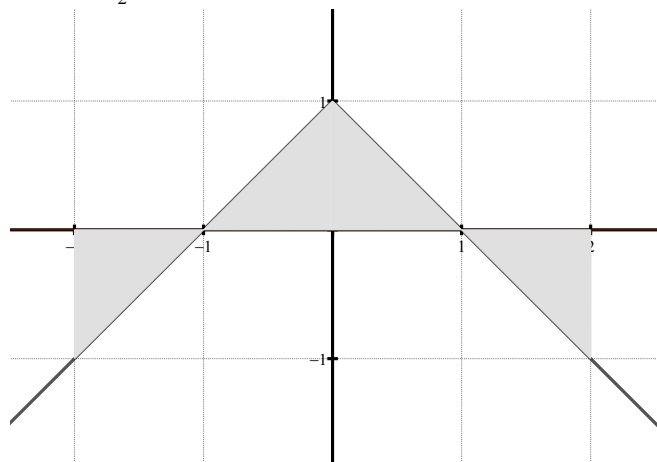


10. If  $\int_a^b f(x) dx > 0$ , then  $f$  is nonnegative for all  $x$  in  $[a, b]$ . FALSE (see below)



11.  $\int_0^2 [x] dx = 1$ .

12. a)  $\int_{-2}^2 (1 - |x|) dx = 0$



b)  $\int_0^3 |3x - 6| dx = 7.5$

