Name KEY Date DUE FRI 10/5 Period______

Worksheet 4.1—Tangent Line Problem

NOTATION | NEWTON | LIEBNITZ
DERIVATIVE | $y', f'(x)$ | $\frac{dy}{dx}, \frac{d}{dx}, \frac{df}{dx}$

Show all work.

1. Find the slope of the tangent lines to the graphs of the following functions at the indicated points.

(a) $f(x) = 3 - 2x$ at $(-1, 5)$

STEP 1 | FIND $\frac{dy}{dx}$

$3-2x-2h$
$-3+2x$

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{x+h-x} = \lim_{h\to 0}\frac{[3-2(x+h)]-[3-2x]}{h}$$

$$\lim_{h\to 0}\frac{-2h}{h} = \boxed{-2}$$

STEP 2 | FIND POINT $(-1, 5)$

STEP 3 | $\boxed{y-5=-2(x+1)}$

(b) $g(x) = 5 - x^2$ at $x = 2$

$g'(x) = -2x$

$g'(2) = -4$

POINT | $g(2) = 5-(2)^2 = 1$

$(2, 1)$

$\boxed{y-1=-4(x-2)}$

2. Find the derivative function, either $f'(x)$ or $\frac{dy}{dx}$, of each of the following using the limit definition.

(a) $f(x) = 2x^2 + 3x - 4$

$\boxed{f'(x) = 4x + 3}$

(b) $y = \frac{3}{x-1}$

$$f'(x) = \lim_{h\to 0}\frac{\frac{(x-1)}{(x-1)}\left[\frac{3}{x+h-1}\right] - \left[\frac{3}{(x-1)}\right]\cdot\frac{(x+h-1)}{(x+h-1)}}{h}$$

$$= \lim_{h\to 0}\frac{\frac{-3h}{(x+h-1)(x-1)}}{h}$$

S.W.
$3x-3$
$-3h-3x+3$

$$= \lim_{h\to 0}\frac{-3\cancel{h}}{(x+h-1)(x-1)}\cdot\frac{1}{\cancel{h}}$$

$$f'(x) = \frac{-3}{(x-1)(x-1)} = \boxed{\frac{-3}{(x-1)^2}}$$

(c) $f(x) = \sqrt{x-2}$

$$\frac{dy}{dx} = \lim_{h\to 0}\frac{[\sqrt{x+h-2}]-[\sqrt{x-2}]}{x+h-x}$$

$$= \lim_{h\to 0}\frac{\sqrt{x+h-2}-\sqrt{x-2}}{h}\cdot\frac{\sqrt{x+h-2}+\sqrt{x-2}}{\sqrt{x+h-2}+\sqrt{x-2}}$$

SW
$x+h-2$
$-(x-2)$
$= h$

$$= \lim_{h\to 0}\frac{\cancel{h}}{\cancel{h}(\sqrt{x+h-2}+\sqrt{x-2})}$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{x+0-2}+\sqrt{x-2}}$$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{x-2}}$$

PERPENDICULAR

3. For each of the following, find the equation of BOTH the tangent line and the $\boxed{\text{normal}}$ line to the function at the indicated points.

(a) $g(x) = x^2 + 1$ at $(2,5)$

STEP 1 $g'(2)$

$$\lim_{h\to 0} \frac{[(2+h)^2+1]-[2^2+1]}{h}$$

S.W.

4+4h+h²
1
−5

$$= \lim_{h\to 0} \frac{h^2+4h}{h}$$

$$= \lim_{h\to 0} h+4$$

$$= 4$$

TANGENT

$y-5=4(x-2)$

NORMAL

$y-5=-\frac{1}{4}(x-2)$

(b) $y = \sqrt{x} - 1$ at $x = 9$

POINT $(1, \sqrt{9}-1)$
$= (1,2)$

$y' = \frac{1}{2\sqrt{x}}$, So $y'(9) = \frac{1}{6}$

TANGENT

$y-2=\frac{1}{6}(x-1)$

NORMAL

$y-2=-6(x-1)$

4. Find an equation of the line that is tangent to $f(x) = x^3$ and parallel to the line $3x - y + 1 = 0$

$f'(x) = 3x^2$

$-y = -3x + 1$
$y = 3x - 1$
* SLOPE MUST BE 3 *

SO WE NEED A POINT

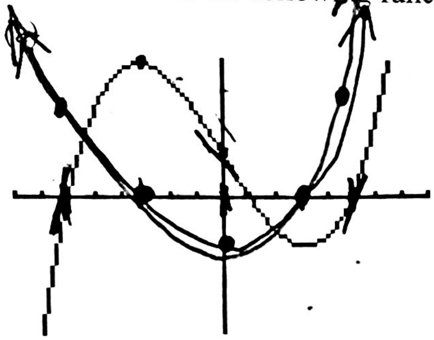
$f'(x) = 3$, THUS $3x^2 = 3 \rightarrow x = \pm 1$

THEREFORE $f'(1) = 3$ AND $f'(-1) = 3$

$\boxed{\text{POINTS}}$ $(1,1)$ AND $(-1,-1)$

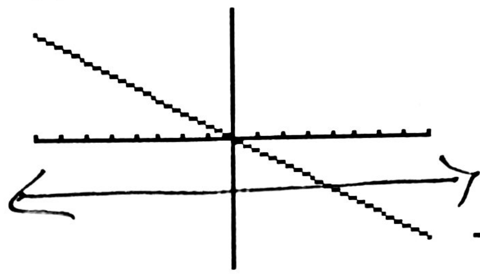
TWO POSSIBLE ANSWERS: $y-1=3(x-1)$
OR
$y+1=3(x+1)$

5. For each of the following functions $f(x)$, on the same coordinate grid, sketch a graph of $f'(x)$.

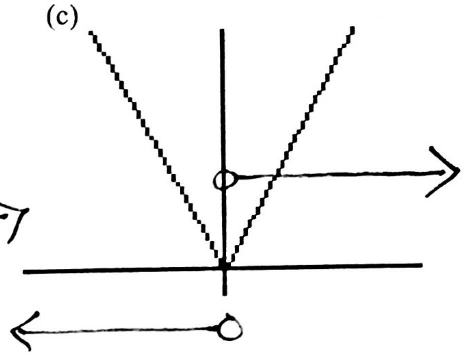


$f'(-6)=3$ $f'(-3)=0$ $f'(0)=-1$ $f'(3)=0$

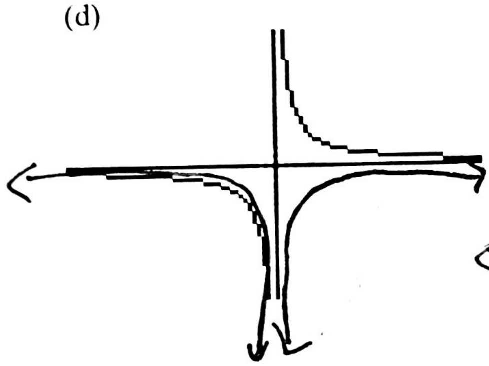
(b)



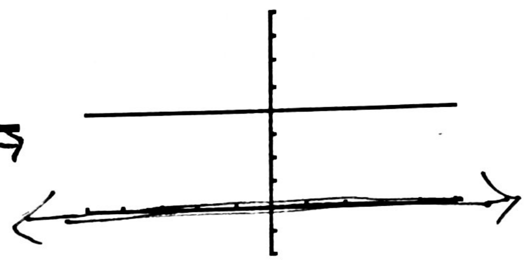
(c)



(d)

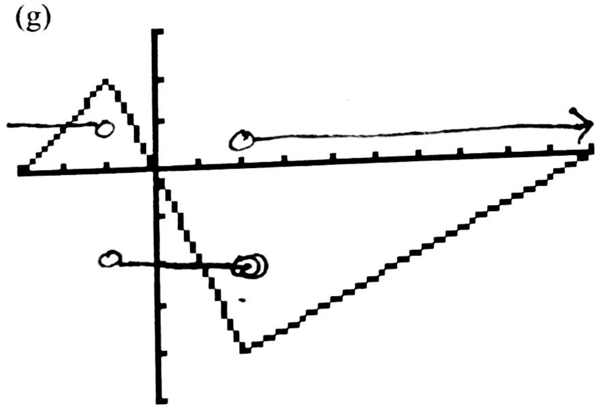


(e)

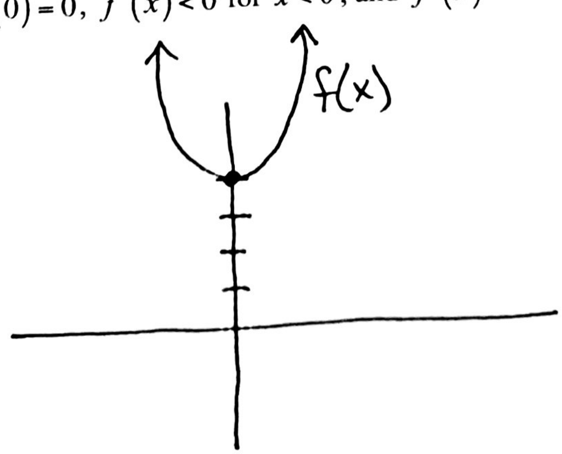


(f)

(g)



6. Sketch a function that has the following characteristics.
$f(0)=4$, $f'(0)=0$, $f'(x)<0$ for $x<0$, and $f'(x)>0$ for $x>0$



$dx = 2\sqrt{x-2}$

ALL WORK ON SEPERATE PAPER

201-103-RE - Calculus 1

WORKSHEET: DEFINITION OF THE DERIVATIVE

1. For each function given below, calculate the **derivative at a point** $f'(a)$ using the limit definition.

   (a) $f(x) = 2x^2 - 3x$ $\quad f'(0) = ?$

   (b) $f(x) = \sqrt{2x+1}$ $\quad f'(4) = ?$

   (c) $f(x) = \dfrac{1}{x-2}$ $\quad f'(3) = ?$

2. For each function $f(x)$ given below, find the **general derivative** $f'(x)$ as a new function by using the limit definition.

   (a) $f(x) = \sqrt{x-4}$ $\quad f'(x) = ?$

   (b) $f(x) = -x^3$ $\quad f'(x) = ?$

   (c) $f(x) = \dfrac{x}{x+1}$ $\quad f'(x) = ?$

   (d) $f(x) = \dfrac{1}{\sqrt{x}}$ $\quad f'(x) = ?$

3. For each function $f(x)$ given below, find the **equation of the tangent line** at the indicated point.

   (a) $f(x) = x - x^2$ at $(2, -2)$

   (b) $f(x) = 1 - 3x^2$ at $(0, 1)$

   (c) $f(x) = \dfrac{1}{2x}$ at $x = 1$

   (d) $f(x) = x + \sqrt{x}$ at $x = 1$

ANSWERS:

1. (a) $f'(0) = -3$ (b) $f'(4) = 1/3$ (c) $f'(3) = -1$
2. (a) $f'(x) = \frac{1}{2\sqrt{x-4}}$ (b) $f'(x) = -3x^2$ (c) $f'(x) = \frac{1}{(x+1)^2}$ (d) $f'(x) = \frac{-1}{2x^{3/2}}$
3. (a) $y = -3x + 4$ (b) $y = 1$ (c) $y = -\frac{1}{2}x + 1$ (d) $y = \frac{3}{2}x + \frac{1}{2}$