① (Opt.)

Prim.

$$D^2 = (x_1 - x_2)^2 + (y_2 - y_1)^2$$

Sec $y = 3 - 2x$

$y = 3 - 2(6/5)$
$= 3/5$

Points

$(2,1)$
$(x, 3-2x)$

$$M = (x-2)^2 + (1 - (3-2x))^2$$
$$M = (x-2)^2 + (-2 + 2x)^2$$
$$M = x^2 - 4x + 4 + 4 - 8x + 4x^2$$
$$M = 5x^2 - 12x + 8$$
$$M' = 10x - 12 \longrightarrow \boxed{x = 6/5}$$

Sentence The point closest to (2,1) which lies on $2x+y=3$ is $(6/5, 3/5)$.

---

② (Opt.)

Primary

$M = x + y$

Sec

$\cancel{y = \left(\frac{2}{x}\right)^2}$

$y = 2 \cdot \frac{1}{x^2}$

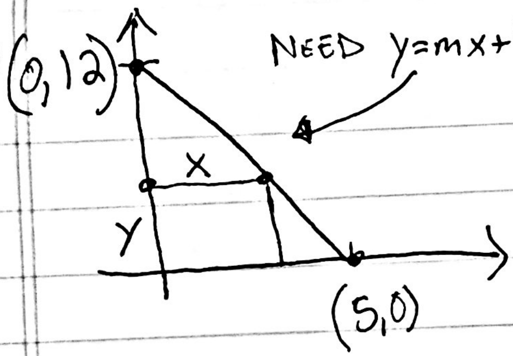
* Reciprocal of x * is $1/x$

$\cancel{M = x + \left(\frac{2}{x}\right)^2 = x + \frac{4}{x^2}}$
$\cancel{M = x + 4x^{-2}}$
$\cancel{M' = 1 - 8x^{-3}}$
$\cancel{0 = 1 - \frac{8}{x^3}}$
$\cancel{8/x^3 = 1}$
$\cancel{8 = x^3}$
$\cancel{\boxed{2 = x}}$

$$M = x + 2x^{-2}$$
$$M' = 1 \cancel{3} - 4x^{-3}$$
$$0 = 1 - 4/x^3$$
$$4/x^3 = 1$$
$$4 = x^3$$
$$\boxed{\sqrt[3]{4} = x}$$

Sentence The positive number is $\sqrt[3]{4}$.

(3)



NEED $y=mx+b \rightarrow y=\frac{-12}{5}x+12$

PRIM	SEC
$M=xy$	$y=\frac{-12}{5}x+12$
	$y=\frac{-12}{5}(2.5)+12=\underline{\underline{6}}$

$$M=x\left(\frac{-12}{5}x+12\right)=\frac{-12}{5}x^2+12x$$
$$M'=-\frac{24}{5}x+12$$
$$0=-{}^{24}\!/_5x+12$$

$$\frac{24}{5}x=12$$
$$\underline{x=2.5}$$

SENTENCE: THE LARGEST RECTANGLE WILL BE 2.5" ~~UNITS~~ BY 6"

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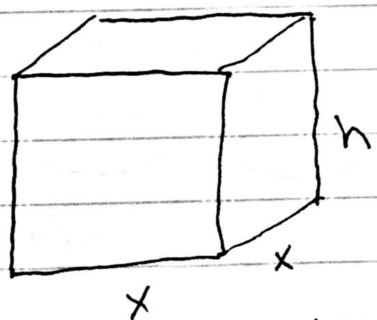
(4)



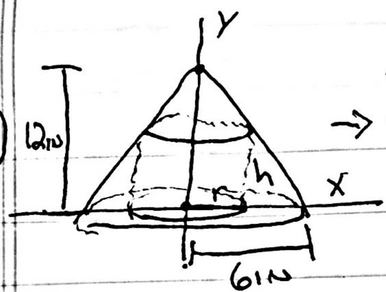
PRIM. (COST!)	SEC
$M=1\cdot(x^2+4xh)+2(x^2)$	$96=x^2\cdot h$
$=3x^2+4xh$	$h=96/x^2$
	$h=96/(4)^2=\underline{\underline{6}}$

$$M=3x^2+4x\left(\frac{96}{x^2}\right)=3x^2+384x^{-1}$$
$$M'=6x-384x^{-2}$$
$$0=6x-\frac{384}{x^2}$$

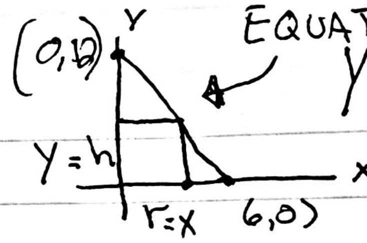
$$\frac{384}{x^2}=6x$$
$$384=6x^3$$
$$64=x^3$$
$$\underline{\underline{4=x}}$$

SENTENCE: THE DIMENSIONS OF THE BOX SHOULD BE 4 IN BY 4 IN BY 6 IN

(5)



$\rightarrow$ TRANSLATE To 2D $\rightarrow$



EQUATION: $y = \frac{-12}{6}x + 12$

PRIM

$V = \pi r^2 h = \pi x^2 y$

SEC

$y = -2x + 12$

$y = -2(4) + 12 = \underline{\underline{4}}$

$M = \pi \cdot x^2(-2x+12) = -2\pi x^3 + 12\pi x^2$

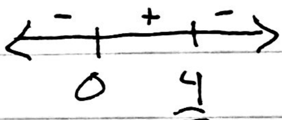
$M' = -6\pi x^2 + \cancel{24}\,\pi x$

$\frac{0}{6\pi} = \frac{-6\pi x^2}{6\pi} + \frac{24\pi x}{6\pi}$

$0 = -x^2 + 4x$

$0 = -x(x-4)$

So $x = 0, 4$



SENTENCE: THE ~~BEAEN~~ CYLINDER WILL HAVE DIMENSIONS $r = h = 4$ WITH A VOLUME OF 64π in^3.

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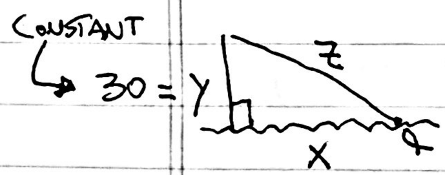
(6) **PIC**



GIVEN: $\frac{dz}{dt} = -2$ ft/sec

FIND: $\frac{dx}{dt}$ WHEN $Z = 50$

$$\frac{d}{dt}\left[30^2 + x^2 = z^2\right]$$

$$= \quad 0 + 2x\frac{dx}{dt} = 2z \cdot \frac{dz}{dt}$$

STILL NEED X... $30^2 + x^2 = 50^2 \Rightarrow X = 40$ WHEN $Z = 50$

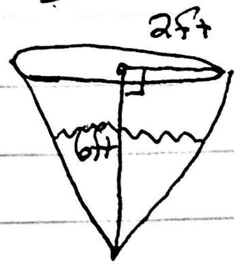
MAGIC MOMENT

$$2(40)(-2) = 2(50)\frac{dz}{dt} \rightarrow \frac{dz}{dt} = -8/5$$

SENTENCE: WHEN THE LINE IS 50 FT, THE FISH IS MOVING THROUGH THE WATER AT A SPEED OF 1.6 ft/sec

(7)

PIC



GIVEN

$\frac{dV}{dt} = 2 \text{ ft}^3/\text{MIN}$

FIND

$\frac{dh}{dt}$ WHEN $V = \frac{\frac{\pi}{3}(2)^2 \cdot 6}{2}$

* HALF FULL

$\frac{r}{h} = \frac{2}{6}$

NEED $\frac{dh}{dt}$, SO SOLVE FOR r

$r = \frac{1}{3}h$

$V = \frac{\pi}{3} r^2 \cdot h$

$V = \frac{\pi}{3}\left(\frac{1}{3}h\right)^2 \cdot h$

$V = \frac{\pi}{27} \cdot h^3$

$\frac{dV}{dt} = \frac{\pi}{9} h^2 \cdot \frac{dh}{dt}$

STILL NEED h WHEN $V = 4\pi$ * (HALF FULL)

$4\pi = \frac{\pi}{27} \cdot h^3$

$\sqrt[3]{108} = h$

MAGIC MOMENT

$2 = \frac{\pi}{9}\left(\sqrt[3]{108}\right)^2 \cdot \frac{dh}{dt}$

$\frac{dh}{dt} \approx .253$

SENTENCE: THE WATER IS RISING AT A RATE OF .253 ft/MIN.

(8)

GIVEN ✓

$\frac{dr}{dt} = 1 \text{ mm/SEC}$

FIND

$\frac{dA}{dt}$ WHEN $r = 4$ cm $= 40$ mm

CONVERT!

$A = \pi r^2$

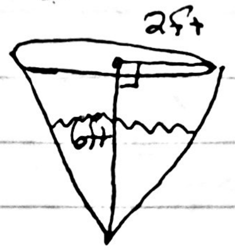
$\frac{dA}{dt} = 2\pi r \cdot \frac{dr}{dt}$

MAGIC MOMENT

$\frac{dA}{dt} = 2\pi(40)(1) = 80\pi$

SENTENCE: THE AREA IS INCREASING BY 80π mm^2/SEC WHEN THE RADIUS IS 4 cm.

(7) PIC)



GIVEN)

$\frac{dV}{dt} = 2\ ft^3/MIN$

FIND)

$\frac{dh}{dt}$ WHEN $V = \frac{\frac{\pi}{3}(2)^2 \cdot 6}{2}$

* HALF FULL

$\frac{r}{h} = \frac{2}{6}$

NEED $\frac{dh}{dt}$, SO SOLVE FOR r

$r = \frac{1}{3}h$

$V = \frac{\pi}{3} r^2 \cdot h$

$V = \frac{\pi}{3}\left(\frac{1}{3}h\right)^2 \cdot h$

$V = \frac{\pi}{27} \cdot h^3$

$\frac{dV}{dt} = \frac{\pi}{9} h^2 \cdot \frac{dh}{dt}$

STILL NEED h WHEN $V = 4\pi$ * (HALF FULL)

$4\pi = \frac{\pi}{27} \cdot h^3$

$\sqrt[3]{108} \approx h$

MAGIC MOMENT

$2 = \frac{\pi}{9}\left(\sqrt[3]{108}\right)^2 \cdot \frac{dh}{dt}$

$\frac{dh}{dt} \approx .253$

SENTENCE: THE WATER IS RISING AT A RATE OF .253 ft/MIN.

(8)

GIVEN)

$\frac{dr}{dt} = 1\ mm/SEC$

FIND)

$\frac{dA}{dt}$ WHEN $r = 4$ cm $= 40$ mm

CONVERT!

$A = \pi r^2$

$\frac{dA}{dt} = 2\pi r \cdot \frac{dr}{dt}$

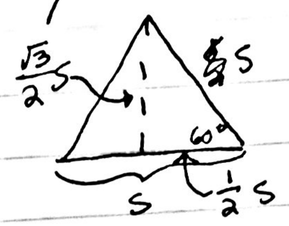
MAGIC MOMENT

$\frac{dA}{dt} = 2\pi(40)(1) = 80\pi$

SENTENCE: THE AREA IS INCREASING BY 80π mm^2/SEC WHEN THE RADIUS IS 4 cm.

PYTHAG. OR SOHCAHTOA!

(9)



$$A = \frac{1}{2}s \cdot \left(\frac{\sqrt{3}}{2}s\right)$$

$$A = \frac{\sqrt{3}}{4}s^2$$

GIVEN

$\frac{ds}{dt} = 1$ mm/SEC

FIND

$\frac{dA}{dt}$ WHEN $A = 14\ mm^2$

$$\frac{dA}{dt} = \frac{\sqrt{3}}{4} \cdot 2s \cdot \frac{ds}{dt} = \frac{\sqrt{3}}{2}s \cdot \frac{ds}{dt}$$

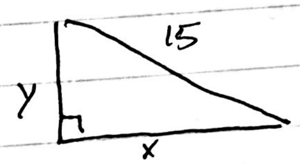
NEED S WHEN A = 14, SO $14 = \frac{\sqrt{3}}{4}s^2 \rightarrow s \approx 5.686$ mm

MAGIC MOMENT

$$\frac{dA}{dt} = \frac{\sqrt{3}}{2}(5.686)\cdot(1) \approx 4.924$$

SENTENCE: THE AREA IS INCREASING AT 4.924 mm^2/SEC WHEN THE AREA IS 14 mm^2

(10)



GIVEN

$\frac{dx}{dt} = -2$ ft/SEC

FIND

$\frac{dy}{dt}$ WHEN $y = 9$

$$\frac{d}{dt}\left[x^2 + y^2 = 15^2\right] \rightarrow 2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

NEED X WHEN $y = 9 \rightarrow 9^2 + x^2 = 15^2 \rightarrow x = 12$

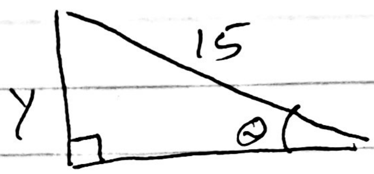
MAGIC MOMENT

$$2(12)(-2) + 2(9)\frac{dy}{dt} = 0 \rightarrow \frac{dy}{dt} = 8/3 \text{ ft/SEC}$$

SENTENCE: THE LADDER IS MOVING AT 8/3 ft/SEC ABOVE THE WALL WHEN THE LADDER IS 9 ft

FROM PREVIOUS

(11)



GIVEN

$\frac{dy}{dt} = \frac{8}{3}$

FIND

$\frac{d\theta}{dt}$ WHEN $y = 9$

$$\frac{d}{dt}\left[\sin\theta = \frac{y}{15}\right]$$

$$\cos\theta \cdot \frac{d\theta}{dt} = \frac{1}{15}\cdot\frac{dy}{dt}$$

NEED $\cos\theta$ WHEN $y = 9$

$$\cos\theta = \frac{12}{15} = \frac{4}{5}$$

MAGIC MOMENT

$$\frac{4}{5}\frac{d\theta}{dt} = \frac{1}{15}\left(\frac{8}{3}\right) \rightarrow \frac{d\theta}{dt} = \frac{2}{9}\ \text{RAD/SEC}$$

SENTENCE: THE ∡ IS INCREASING AT A RATE OF $\frac{2}{9}$ RAD/SEC.

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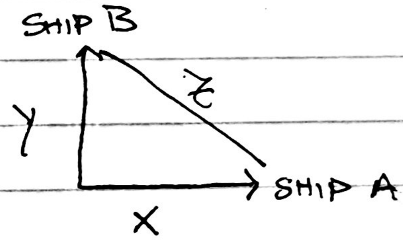
(12)



GIVEN

$\frac{dx}{dt} = 15\text{ mph}$

$\frac{dy}{dt} = 10\text{ mph}$

FIND

$\frac{dz}{dt}$ @ 3:00 am

$$\frac{d}{dt}\left[x^2 + y^2 = z^2\right]$$

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$$

STILL NEED $x, y, + z$ @ 3:00 am

LEFT @ 1:00 am ($D = rt$)

$x = 15(2) = 30$

$y = 10(1) = 10$

LEFT @ 2:00 am

$x^2 + y^2 = z^2$

$30^2 + 10^2 = z^2$

$1000 = z^2$

$\sqrt{1000} = z$

MAGIC MOMENT

$$2(30)(15) + 2(10)(10) = 2\left(\sqrt{1000}\right)\frac{dz}{dt}$$

SENTENCE: THE DISTANCE OF THE SHIPS IS INCREASING BY 17.4 KNOTS (MPH)