IN CLASS OR HW?

Evaluating Definite Integrals with Geometry

Sketch the region whose area is given by the definite integral. Use geometry to evaluate the integral, then check your answer with your calculator. Note: Assume **k > 0** and **r > 0**.

Examples:

1. $\int_{-4}^{3} 7\,dx = 28$
2. $\int_{1}^{3} 2x\,dx = 10$
3. $\int_{0}^{3} (-2x+6)\,dx = 18$
4. $\int_{-4}^{4} (4-|x|)\,dx$ 16
5. $\int_{0}^{3} \sqrt{9-x^2}\,dx = \frac{9}{4}\pi$

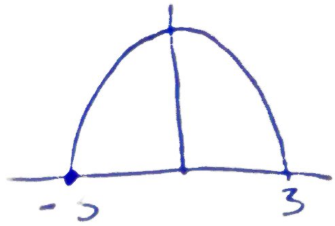
A new geometric formula!

Archimedes discovered that the area of a **parabolic arch** can be found using the formula

$$\text{Area} = \frac{2}{3} \times \text{base} \times \text{height}$$

Find the area of the region bounded by **f(x) = 9 - x²** and the x-axis.

$\frac{2}{3} \cdot 6 \cdot 9 = \boxed{36}$



Problems:

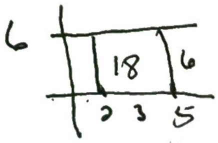
6. $\int_{-5}^{5} 4\,dx = 40$
7. $\int_{a}^{b} k\,dx = k(b-a)$
8. $\int_{2}^{10} \frac{1}{2}x\,dx = 24$
9. $\int_{2}^{12} (8-\frac{1}{2}x)\,dx = 45$
10. $\int_{-4}^{3} (|x|+2)\,dx = 21$
11. $\int_{-4}^{3} |x+2|\,dx = 13$
12. $\int_{-2}^{2} \sqrt{4-x^2}\,dx = 2\pi$
13. $\int_{-k}^{k} (k-|x|)\,dx = k^2$
14. $\int_{0}^{k} kx\,dx$ $\frac{k^3}{2}$
15. $\int_{0}^{r} \sqrt{r^2-x^2}\,dx$ $\frac{1}{4}\pi r^2$
16. $\int_{0}^{6} (6x-x^2)\,dx = 36$
17. $\int_{-\sqrt{k}}^{\sqrt{k}} (k-x^2)\,dx = \frac{4}{3}k^2$

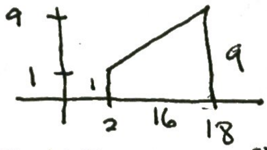
Name ______________________________ Date ______________

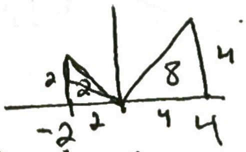
24−13.5 $\boxed{5}$

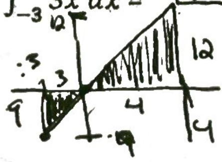
Solve the following:

1) $\int_2^5 6\,dx =$ 18 2) $\int_2^{18} \frac{x}{2}\,dx =$ 80 3) $\int_{-2}^{4} |x|\,dx =$ 10 4) $\int_{-3}^{4} 3x\,dx =$ $\boxed{-10.5}$

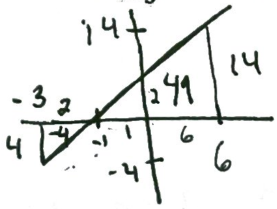


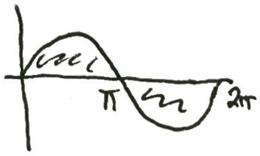


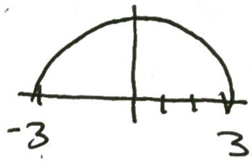




5) $\int_{-3}^{6} (2x+2)\,dx =$ 45 6) $\int_0^{2\pi} \sin x\,dx = 0$ 7) $\int_{-3}^{3} \sqrt{9-x^2}\,dx = \frac{9\pi}{2}$



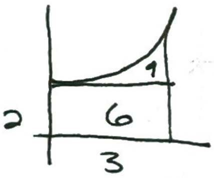


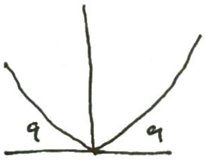


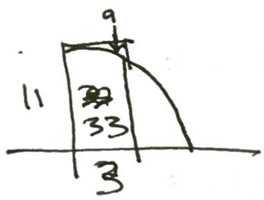
$A = \pi r^2$

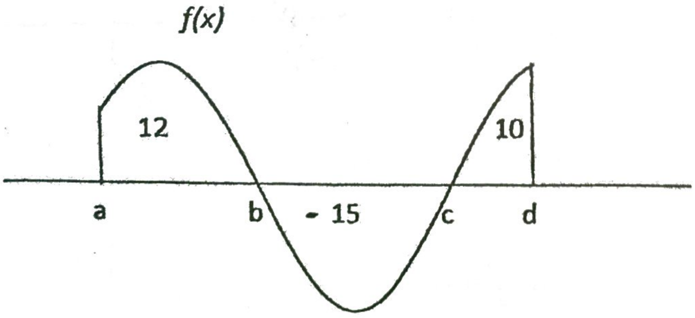
Given: $\int_0^3 x^2\,dx = 9$ find the following:

8) $\int_0^3 (x^2+2)\,dx =$ 15 9) $\int_{-3}^{3} x^2\,dx =$ 18 10) $\int_0^3 (11-x^2)\,dx =$ 33 − 9 = 24









11) $\int_a^b f(x)\,dx =$ 12 12) $\int_a^c f(x)\,dx =$ −3 13) $\int_a^d f(x)\,dx =$ 7 14) $\int_d^b f(x)\,dx =$ −5

12−15 12−15+10 −15+10