Extra Practice

Date_____ Period____

For each problem, find the: x and y intercepts, x-coordinates of the critical points, open intervals where the function is increasing and decreasing, x-coordinates of the inflection points, open intervals where the function is concave up and concave down, and relative minima and maxima. Using this information, sketch the graph of the function.

1) $y = -\dfrac{x^3}{x^2 - 1}$

2) $y = \dfrac{9}{x^2 + 3}$

3) $y = -\dfrac{x}{x^2 - 1}$

4) $y = -\dfrac{x^3}{3} + \dfrac{2x^2}{3}$

5) $y = -\dfrac{x^3}{12} + \dfrac{x^2}{4}$

6) $y = \dfrac{x^3}{3} + \dfrac{2x^2}{3}$

Evaluate each limit. Use L'Hôpital's Rule if it can be applied. If it cannot be applied, evaluate using another method and write a * next to your answer.

7) $\lim\limits_{x \to 0} \dfrac{\sin(3x)}{x}$

8) $\lim\limits_{x \to 0} \dfrac{e^{4x} - 1}{4x}$

9) $\lim\limits_{x \to 0} \dfrac{x^2}{e^x - x}$

10) $\lim\limits_{x \to \infty} \dfrac{e^x}{2x^2}$

For each problem, find all points of absolute minima and maxima on the given interval.

11) $y = \dfrac{x^2}{2} - 4x + 2$; [5, 7]

12) $y = -2x^2 + 12x - 14$; [1, 4]

For each problem, determine if the Mean Value Theorem can be applied. If it can, find all values of *c* that satisfy the theorem. If it cannot, explain why not.

13) $y = x^3 - 2x^2 + 1$; [−1, 2]

14) $y = -x^2 + 6x - 7$; [1, 5]

15) $y = \dfrac{-x^2 + 1}{4x}$; [1, 4]

16) $y = \dfrac{x^2}{4x + 4}$; [−2, 0]

For each problem, find the open intervals where the function is concave up and concave down.

17) $y = -x^3 + x^2 - 4$

18) $y = -\dfrac{x^2}{2x + 2}$

19) $y = -\dfrac{3x}{x - 1}$

20) $y = x^4 - 2x^2 - 1$

For each problem, find the open intervals where the function is increasing and decreasing.

21) $y = \dfrac{x^2}{2} - x + \dfrac{3}{2}$

22) $y = -x^3 + 3x^2 - 5$

For each problem, find all points of relative minima and maxima.

23) $y = \dfrac{1}{x^2 - 4}$

24) $y = \dfrac{x^2}{2} - 2x + 2$

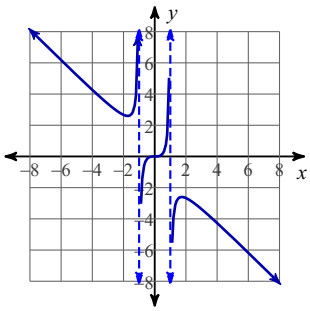
For each problem, find the values of *c* that satisfy Rolle's Theorem.

25) $y = -\dfrac{x^2}{2} - 4x - 8$; [−6, −2]

26) $y = 2x^2 + 16x + 32$; [−5, −3]

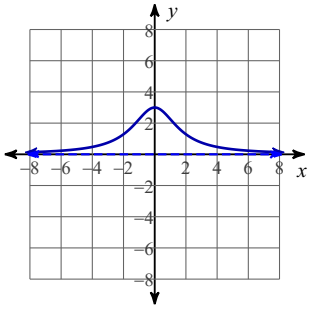
Answers to Extra Practice (ID: 1)

1)



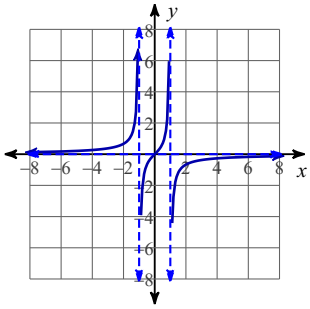
x-intercept at $x = 0$ y-intercept at $y = 0$
Critical points at: $x = -\sqrt{3}, 0, \sqrt{3}$
Increasing: $(-\sqrt{3}, -1), (-1, 1), (1, \sqrt{3})$ Decreasing: $(-\infty, -\sqrt{3}), (\sqrt{3}, \infty)$
Inflection point at: $x = 0$
Concave up: $(-\infty, -1), (0, 1)$ Concave down: $(-1, 0), (1, \infty)$
Relative minimum: $\left(-\sqrt{3}, \frac{3\sqrt{3}}{2}\right)$ Relative maximum: $\left(\sqrt{3}, -\frac{3\sqrt{3}}{2}\right)$

2)



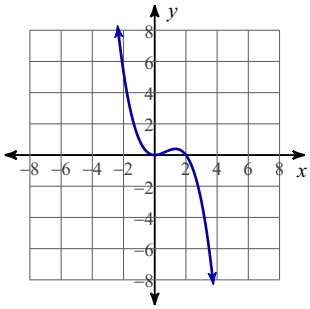
No x-intercepts. y-intercept at $y = 3$
Critical point at: $x = 0$
Increasing: $(-\infty, 0)$ Decreasing: $(0, \infty)$
Inflection points at: $x = -1, 1$
Concave up: $(-\infty, -1), (1, \infty)$ Concave down: $(-1, 1)$
No relative minima. Relative maximum: $(0, 3)$

3)



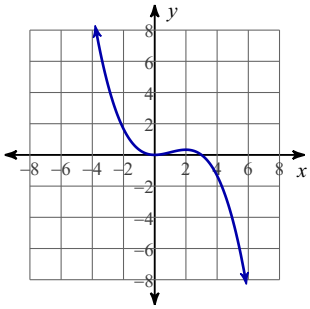
x-intercept at $x = 0$ y-intercept at $y = 0$
No critical points exist.
Increasing: $(-\infty, -1), (-1, 1), (1, \infty)$ Decreasing: No intervals exist.
Inflection point at: $x = 0$
Concave up: $(-\infty, -1), (0, 1)$ Concave down: $(-1, 0), (1, \infty)$
No relative minima. No relative maxima.

4)



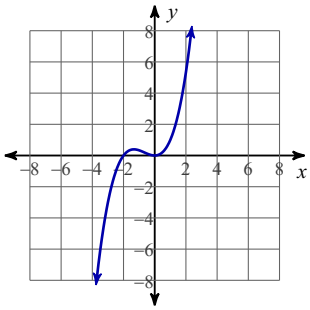
x-intercepts at $x = 0, 2$ y-intercept at $y = 0$
Critical points at: $x = 0, \frac{4}{3}$
Increasing: $\left(0, \frac{4}{3}\right)$ Decreasing: $(-\infty, 0), \left(\frac{4}{3}, \infty\right)$
Inflection point at: $x = \frac{2}{3}$
Concave up: $\left(-\infty, \frac{2}{3}\right)$ Concave down: $\left(\frac{2}{3}, \infty\right)$
Relative minimum: $(0, 0)$ Relative maximum: $\left(\frac{4}{3}, \frac{32}{81}\right)$

5)



x-intercepts at $x = 0, 3$ y-intercept at $y = 0$
Critical points at: $x = 0, 2$
Increasing: $(0, 2)$ Decreasing: $(-\infty, 0), (2, \infty)$
Inflection point at: $x = 1$
Concave up: $(-\infty, 1)$ Concave down: $(1, \infty)$
Relative minimum: $(0, 0)$ Relative maximum: $\left(2, \frac{1}{3}\right)$

6)



x-intercepts at $x = -2, 0$ y-intercept at $y = 0$

Critical points at: $x = -\frac{4}{3}, 0$

Increasing: $\left(-\infty, -\frac{4}{3}\right), (0, \infty)$ Decreasing: $\left(-\frac{4}{3}, 0\right)$

Inflection point at: $x = -\frac{2}{3}$

Concave up: $\left(-\frac{2}{3}, \infty\right)$ Concave down: $\left(-\infty, -\frac{2}{3}\right)$

Relative minimum: $(0, 0)$ Relative maximum: $\left(-\frac{4}{3}, \frac{32}{81}\right)$

7) 3

8) 1

9) 0 *

10) ∞

11) Absolute minimum: $\left(5, -\frac{11}{2}\right)$

Absolute maximum: $\left(7, -\frac{3}{2}\right)$

12) Absolute minimum: $(1, -4)$

Absolute maximum: $(3, 4)$

13) $\left\{\frac{2+\sqrt{7}}{3}, \frac{2-\sqrt{7}}{3}\right\}$

14) $\{3\}$

15) $\{2\}$

16) The function is not continuous on $[-2, 0]$

17) Concave up: $\left(-\infty, \frac{1}{3}\right)$ Concave down: $\left(\frac{1}{3}, \infty\right)$

18) Concave up: $(-\infty, -1)$ Concave down: $(-1, \infty)$

19) Concave up: $(-\infty, 1)$ Concave down: $(1, \infty)$

20) Concave up: $\left(-\infty, -\frac{\sqrt{3}}{3}\right), \left(\frac{\sqrt{3}}{3}, \infty\right)$ Concave down: $\left(-\frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3}\right)$

21) Increasing: $(1, \infty)$ Decreasing: $(-\infty, 1)$

22) Increasing: $(0, 2)$ Decreasing: $(-\infty, 0), (2, \infty)$

23) No relative minima.

Relative maximum: $\left(0, -\frac{1}{4}\right)$

24) Relative minimum: $(2, 0)$

No relative maxima.

25) $\{-4\}$

26) $\{-4\}$